MOCK EXAM 4

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

(2 1/4 hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{m^{-15}n^{9}}{m^{6}n^{-6}}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $\dfrac{2a+b}{2a-b+4}=5$. (3 marks)

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3. Factorize

(a) $4m^2 + 20mn + 25n^2$,

(b) $6m + 15n – 4m^2 – 20mn – 25n^2$. (3 marks)

4. In a school, S6 has 5 classes. Each class has the same number of students. In each class, there are 3 more girls than boys. There are 175 students in S6. Find the number of girls in S6. (4 marks)

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5. (a) Solve the inequality $\frac{5x+3}{2} < 9x + 8$.

(b) How many integers satisfy both the inequalities $\frac{5x+3}{2} < 9x + 8$ and $12 - 3x \geq 0$?

(4 marks)

6. The marked price of a jacket is \$300. The jacket is sold at a discount of 30% on its marked price and the percentage profit is 40%.

(a) Find the selling price of the jacket.

(b) Find the cost of the jacket. (4 marks)

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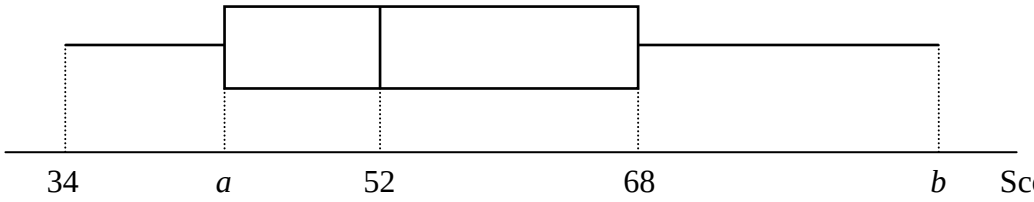
7. The box-and-whisker diagram below shows the distribution of scores got by a large group of students taking a mathematics test:



The inter-quartile range and the range of the distribution are 25 marks and 55 marks respectively.

(a) Find a and b.

(b) The students join a training course. It is found that the lowest marks got by the students in another mathematics test after the training course is 10 marks higher than that before the training. The course organizer claims that at least 25% of the students show improvement in the mathematics test after the training. Do you agree? Explain your answer. (4 marks)

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8. Let $f(x) = 6x^3 + 3x^2 + cx + 4$ where c is a constant. When f(x) is divided by x + 2, the remainder is –10.

   (a) Is 2x – 1 a factor of f(x)? Explain your answer.

   (b) Someone claims that all roots of the equation f(x) = 0 are rational numbers. Do you agree? Explain your answer. (5 marks)

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9. In Figure 1, AB, BC, CD and AD are chords of the circle. AC and BD intersect at E. It is given that AD = BC.

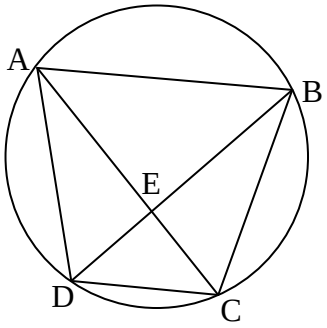

Figure 1

(a) Prove that $\triangle ADE \cong \triangle BCE$.

(b) If BD = 23 cm, CE = 8 cm and AD = 17 cm, are AC and BD perpendicular to each other? Explain your answer. (5 marks)

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SECTION A(2) (35 marks)

10. Let $C be the cost of making a ball of surface area A cm^2. It is given that C is the sum of two parts, one part is a constant and the other part varies as A. When A = 35, C = 153 and when A = 41, C = 177.

    (a) Find the cost of making a ball of surface area 14 cm^2. (4 marks)

    (b) If the volume of a larger ball is 8 times that of the ball in (a), find the cost of making this larger ball. (2 marks)

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11. The heights of twenty students (in cm) in a group are shown as follows:

123	124	125	126	126	128	129	130	131	131
131	133	133	134	134	135	136	136	137	138

(a) Find the mean and mode of the above distribution. (2 marks)

(b) Four more students join the group. The mean of the heights of these four students is 125 cm. It is found that the range of the distribution doesn't change.

(i) Write down the mean of the heights of all students now.

(ii) Is it possible that the mode of the distribution is higher than that found in (a)? Explain your answer. (4 marks)

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12. 3 identical solid right circular cones of base radius R cm and height H cm are melted and recast into 32 smaller identical solid right circular cones of base radius r cm and height h cm. It is given that H : h = 27 : 8.

(a) Find r : R. (2 marks)

(b) It is given that r = 18 cm and H = 81 cm.

(i) Find the curved surface area of a small right circular cone.

(ii) A craftsman claims that the total curved surface area of the smaller right circular cones is more than 2 times the total curved surface area of the larger right circular cones. Do you agree? Explain your answer. (5 marks)

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13. Figure 2 shows the graphs for Amy and Bob cycling on the same straight road between town P and town Q during the period 9:15 to 11:00 in a morning. Bob cycles at a constant speed. Town P and town Q are 28 km apart.

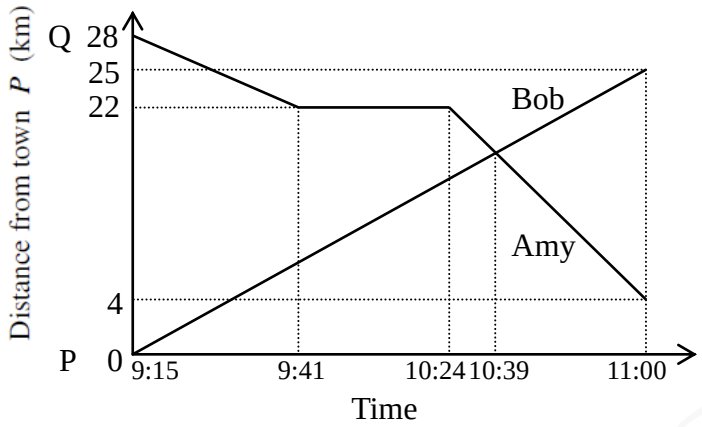

Figure 2

(a) How long does Amy rest during the period? (2 marks)

(b) How far from town Q do Amy and Bob meet during the period? (3 marks)

(c) Bob claims that by keeping the same speed, he will reach town Q before Amy reaches town P. Do you agree? Explain your answer. (2 marks)

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14. The x-intercept and y-intercept of straight line L_1 are –3 and 6 respectively. The equation of the straight line L_2 is $x + 2y – 17 = 0$. L_1 intersects with L_2 at A.

(a) Find the equation of L_1. (2 marks)

(b) Find the coordinates of A. (1 mark)

(c) L_1 and L_2 cuts the x-axis at B and C respectively.

(i) Find the equation of the circle passing through A, B and C.

(ii) P is a moving point in the rectangular coordinate plane such that AP = BP. Denote the locus of P by Γ.

(1) Describe the geometric relationship between Γ and AB.

(2) If Γ cuts AB and the x-axis at D and E respectively, find the area of $\triangle$BDE.

(6 marks)

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SECTION B (35 marks)

15. The graph in Figure 3 shows the linear relation between $\log_{16} x$ and $\log_8 y$. The intercepts on the horizontal axis and the vertical axis of the graph are –3 and 2 respectively. Express the relation between x and y in the form $y = Ax^k$, where A and k are constants. (3 marks)

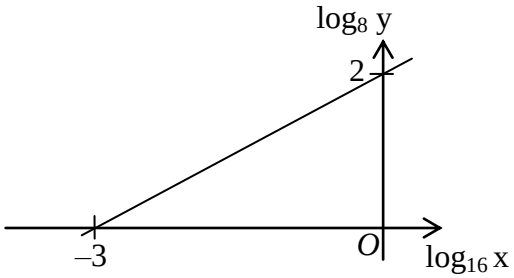

Figure 3

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16. There are 10 students from class A and 14 students from class B. 6 students are randomly selected to form a volleyball team.

(a) Find the probability that there are equal numbers of students from both classes in the team. (2 marks)

(b) Find the probability that at least 1 student come from class A. (2 marks)

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17. Figure 4 shows a solid pyramid VABCD with a square base, where AB = 20 cm, VA = VC = 30 cm and $\angle VAB = 80^o$.

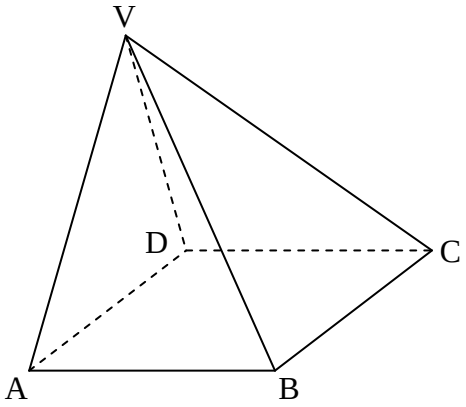

Figure 4

(a) Find the angle between the plane VAB and the plane VBC. (5 marks)

(b) Let P be a movable point on the slant height VB. Describe how $\angle APC$ varies as P moves from V to B. Explain your answer. (2 marks)

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18. The coordinates of the centre of the circle C are (5, 2). It is given that the straight line y = 6 is a tangent to C.

(a) Find the equation of C. (2 marks)

(b) The slope and y-intercept of the straight line L is 1 and k respectively. If L cuts C at A and B, find

(i) the coordinates of the mid-point of AB in terms of k and

(ii) the equation of the locus of the mid-point of AB as k varies. (6 marks)

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19. Bob invests $800 000 in a retirement scheme provided by a trust company at the end of a certain year. The trust company guarantees that the growth of the amount at the end of each year is r% of the total amount at the end of the previous year. At the end of each year, the company charges $6 000 as the management fee. The fee is deducted from the total amount at the end of each year. The company starts to charge the fee at the end of the 2nd year. It is found that the total amount is $869 700 at the end of the 3rd year.

(a) (i) Express, in terms of r, the total amount at the end of the 2nd year.

(ii) Find r. (4 marks)

(b) (i) Express, in terms of n, the total amount at the end of the nth year.

(ii) At the end of which year will the total amount exceed $2 000 000? (5 marks)

(c) It is assumed that the amount needed by Bob for the coming 15 years if he retires at the end of the mth year is $\$a(1.1025)^m + b$, where a and b are constants. It is found that for m = 1, the amount is $400 000. For m = 2, the amount is $410 250. Based on the above assumption, the trust company claims that the total amount of the investment will be greater than the amount needed by Bob for the coming 15 years if he retires at the end of any certain year. Is the claim correct? Explain your answer. (4 marks)

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