MOCK EXAM 1

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

$(2 \frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{2}{3x+5} + \dfrac{6}{2-9x}$. (3 marks)

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2. Factorize

   (a) $4x^2 + 20xy + 25y^2$,

   (b) $4x^2 + 20xy + 25y^2 - 8x - 20y$. (3 marks)

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3. Make x the subject of the formula $Ax = (6x - 5B)C$. (3 marks)

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4. Consider the compound inequality

$$4(x - 5) \leq 35 - x \text{ or } x > 6 \quad \ldots\ldots (*).$$

(a) Solve (*).

(b) Write down the smallest positive integer satisfying (*). (4 marks)

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5. Let a, b and c be non-zero numbers such that $5a = 2c$ and $\dfrac{5a+4b}{a+3b} = 2$. Find $\dfrac{9a+4b}{6c-10a}$.

(4 marks)

6. The selling price of a bag is 30% higher than a jacket while the selling price of the jacket is 30% lower than a pair of shoes. It is given that the selling price of the jacket is \$280.

   (a) Find the selling price of the bag.

   (b) Which one has the highest selling price? Explain your answer. (4 marks)

7. In a polar coordinate system, O is the pole. The polar coordinates of the points A, B and C are $(7, 134^\circ)$, $(24, 224^\circ)$ and $(70, 314^\circ)$ respectively.

   (a) Find $\angle AOB$.

   (b) Are A, O and C collinear?

   (c) Find the perimeter of $\triangle ABC$

   (4 marks)

8. In Figure 1, *E* is the point of intersection of *AC* and *BD*. It is given that $\angle ACB = \angle ADB = 90^\circ$ and *AD* = *BC*.

Figure 1
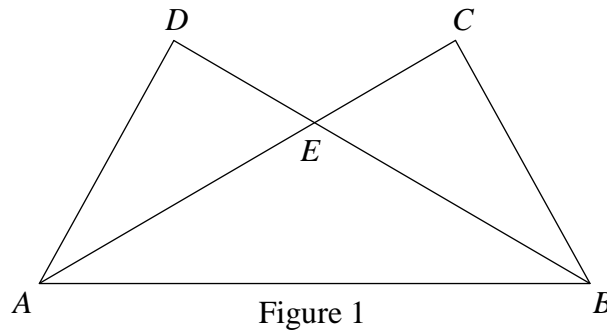
(a) Prove that $\triangle ABC \cong \triangle BAD$.

(b) If *AD* = 80 cm and *DE* = 60 cm, find the area of $\triangle AEB$.

(5 marks)

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Answers written in the margins will not be marked.

9. The table below shows the distribution of the number of pens owned by a class of students

Number of pens	4	5	6	7	8	9
Number of students	4	16	9	10	k	2

If a student is randomly selected from the class, then the probability that the student owns more than

5 pens is $\frac{5}{9}$.

(a) Find k.

(b) Write down the mean, the median and the mode of the distribution.

(5 marks)

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SECTION A(2) (35 marks)

10. It is given that f(x) is partly constant and partly varies as x. Suppose that f(-2) = -16 and f(5) = 12.

    (a) Find f(x). (3 marks)

    (b) Let g(x) = xf(x) + k, where k is a real constant. If all the roots of the equation g(x) = 0 are real numbers, find the range of values of k. (3 marks)

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11. The heights of a group of students (in cm) are shown below:

141	142	144	145	146	150	152	155	158	162
164	166	168	170	170	171	173	174	174	175

(a) Write down the mean, the median and the range of the heights of the students. (3 marks)

(b) Two students join the group. It is found that the mean of the heights of the new group is decreased by 1 cm while the range of the heights of the new group is increased by 1 cm. Find the possible heights of the two students. (4 marks)

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12. Denote the origin by O.

   (a) A and B are points lying on the positive y-axis such that the y-coordinate of A is greater than the y-coordinate of B. A horizontal line which passes through B cuts the straight line $y = mx$ at the point C such that $AB = BC$, where m is positive constant. Let D be a point such that $ABCD$ is a square. Express the slope of OD in terms of m. (3 marks)

   (b) The coordinates of points M and N are $(2, 6)$ and $(0, 10)$ respectively. Let P and Q be points lying OM and MN respectively while R and S be points lying on the y-axis. If the quadrilateral $PQRS$ is a square, find the y-coordinate of P. (4 marks)

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13. Figure 2 shows a frustum with the height of 8 cm. The radii of the upper surface and lower surface are in the ratio of 3 : 5 and the volume of the frustum is 1176π cm^3.
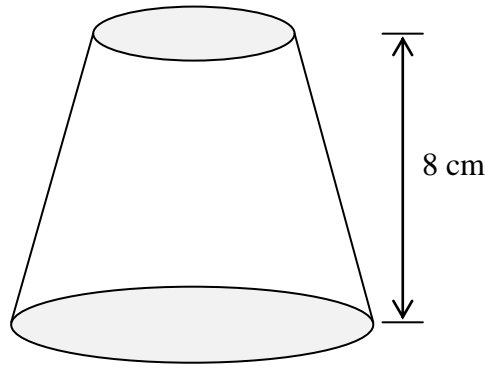
Figure 2

(a) Find the radius of the upper surface. (3 marks)

(b) Someone claims that the curved surface area of the frustum is larger than 250π cm^2. Do you agree? Explain your answer. (4 marks)

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Answers written in the margins will not be marked.

14. Let $f(x) = (5x^2 + 2x + p)(qx^2 + rx - 3)$, where p, q and r are constants. The constant term of $f(x)$ is -9.

    (a) Write down the value of p.   (1 mark)

    (b) It is given that $x - 3$ is a factor of $f(x)$. When $f(x)$ is divided by $x - 1$, the remainder is -60.

        (i) Find q and r.

        (i) How many real roots does the equation $f(x) = 0$ have? Explain your answer.   (7 marks)

SECTION B (35 marks)

15. It is given that $\log_8 y$ is a linear function of $\log_4 x$. Denote the graph of the linear function by L. The slope of L is 6 and L passes through (4, 25). Express y in terms of x. (3 marks)

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16. There are 5 English books, 4 Chinese books and 6 comics on a book shelf. 5 books are randomly selected from these books.

    (a) Find the probability that exactly 2 comics are selected. (2 marks)

    (b) Find the probability that at least 2 comics are selected. (2 marks)

Answers written in the margins will not be marked.

17. The coordinates of the points Q and R are $(1, 2)$ and $(4, -1)$ respectively.

    (a) Let P be a moving point in the rectangular coordinate plane such that $PQ = PR$. Denote the locus of P by Γ.

        (i) Describe the geometric relationship between Γ and QR.

        (ii) Find the equation of Γ. (3 marks)

    (b) Let C be the circle which passes through Q, R and the point $(-8, -7)$.

        (i) Find the equation of C.

        (ii) The coordinates of the point U are $(-10, -12)$. It is found that U lies outside C. UV and UW are the tangents to C at the points V and W respectively. Is the area of the circumcircle of $\triangle UVW$ greater than 100? Explain your answer. (5 marks)

Answers written in the margins will not be marked.

18. Figure 3(a) shows a piece of triangular paper card *ABC* with *AC* = 30 cm, $\angle CAB = 50^\circ$ and $\angle CBA = 35^\circ$. Let *D* be a point lying on *AB* such that $\angle CDA = 80^\circ$.
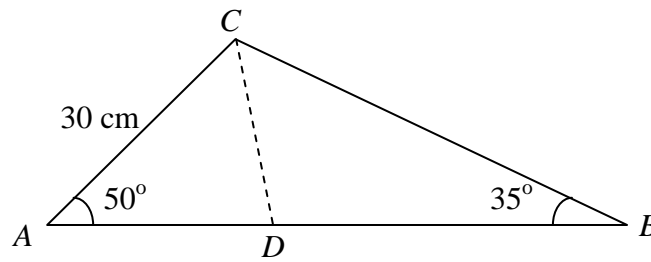
Figure 3(a)

(a) Find *BC* and *BD*. (4 marks)

(b) The triangular paper card in Figure 3(a) is folded along *CD* such that $\triangle CDB$ lies on the horizontal plane as shown in Figure 3(b). It is given that $\angle ACB = 72^\circ$.
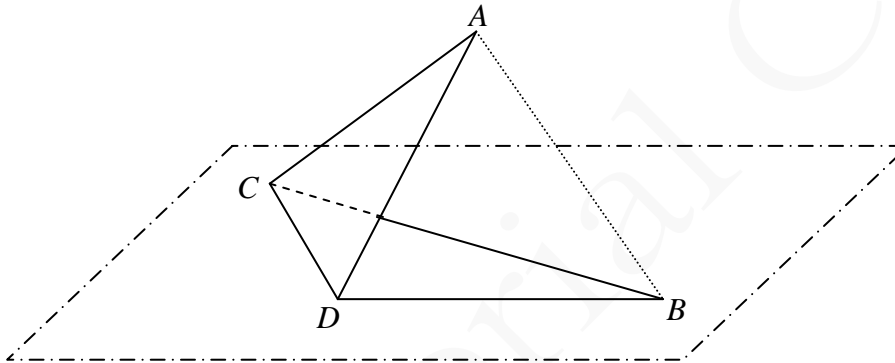

Figure 3(b)

(i) Find the distance between *A* and *B*.

(ii) Someone claims that as the angle between plane *ACD* and plane *BCD* varies, the volume of the tetrahedron *ABCD* cannot exceed 2500 cm^3. Do you agree? Explain your answer.

(4 marks)

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Answers written in the margins will not be marked.

19. Let $f(x) = 3x^2 + 6mx - 12x + 3m^2 - 12m + n$, where m and n are real constants such that $mn < 0$. Denote the vertex of the graph of $y = f(x)$ by P.

(a) Using the method of completing the square, express the coordinates of P in terms of m and n.

(2 marks)

(b) Describe the geometric meaning represented by the transforming $f(x)$ to $4f(x - 16)$. (2 marks)

(c) Denote the vertex of the graph of $y = 4f(x - 16)$ by Q. Let (a_1, b_1) and (a_2, b_2) be the coordinates of P and Q respectively. It is given that a_1, $n - 1$, a_2 is geometric sequence and b_1, $3 - m$, b_2 is an arithmetic sequence.

(i) Find the coordinates of P and Q.

(ii) The coordinates of the points R and S are $(1 - 4t, t + 9)$ and $(3 - 2t, 2t + 6)$ respectively, where t is a real number. Is it possible that $PQRS$ is a rhombus? Explain your answer.

(8 marks)

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