MOCK EXAM 3

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

($2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

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SECTION A(1) (35 marks)

1. Simplify $\dfrac{x^9y^7}{(x^2y^3)^4}$ and express your answer with positive indices. (3 marks)

2. Factorize

   (a) $x^2 + 5x - 6$,

   (b) $x^4 + 5x^3 - 6x^2$ (3 marks)

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3. The table below shows the distribution of the numbers of children in some families.

Number of children	0	1	2	3	4
Number of families	5	20	17	6	2

Find the median, the mode and the standard deviation of the above distribution. (3 marks)

4. Consider the formula $3(x - 2y) = 6x - 4$.

(a) Make y the subject of the above formula.

(b) If the value of x is increased by 4, write down the change in the value of y. (4 marks)

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5. The price of 5 rulers and 6 pens is \$73 while the price of 7 rulers and 9 pens is \$107. Find the price of a pen. (4 marks)

6. In a polar coordinate system, O is the pole. The polar coordinates of A, B and C are $(16, 59^{\circ})$, $(12, 239^{\circ})$ and $(11, 329^{\circ})$ respectively.

   (a) Describe the geometric relationship between OC and AB.

   (b) Find the area of ΔABC. (4 marks)

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7. In Figure 1, D is a point lying on BC such that $\angle DAC = \angle ABD$.

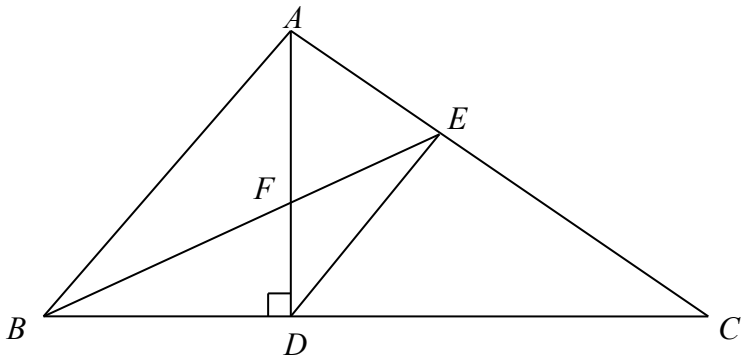

Figure 1

(a) Prove that $\triangle ABC \sim \triangle DAC$.

(b) If $\angle DEC = 90^{\circ}$,

(i) Write down all the triangle(s) which is/are similar to $\triangle ABC$ other than $\triangle DAC$.

(ii) Write down another pair of similar triangles other than those in (i). (4 marks)

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8. There are 8 cards numbered 6, 7, 8, 8, 10, 12, 13 and k respectively where k is a positive integer. It is given that the median is 8.5.

   (a) Find the value of k.

   (b) Two cards are drawn randomly from the cards one by one without replacement. Find the probability that their product is a multiple of 3. (5 marks)

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9. In Figure 2, the volume of the solid right prism *ABCDEFGH* is 1 440 cm^3. The base *ABCD* of the prism is a rhombus. It is given that $AC = 16$ cm and $DE = 15$ cm.

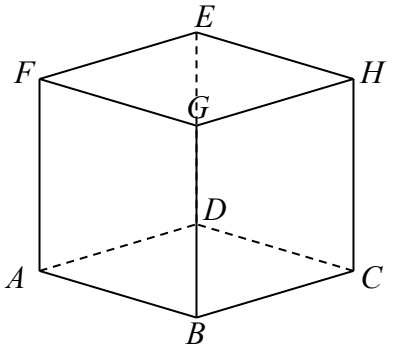

Figure 2

Find

(a) the length of *BD*,

(b) the total surface area of the prism *ABCDEFGH*. (5 marks)

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SECTION A(2) (35 marks)

10. The stem-and-leaf diagram below shows the distribution of the numbers of weekly hours spent by a group of students in doing homework:

Stem (tens)	Leaf (units)											
0	7	7	9	9								
1	0	1	2	2	4	6	6	6	6	7	7	8	9
2	0	a										

It is given that the range of the above distribution is the triple of its inter-quartile range.

(a) Find a. (4 marks)

(b) If a student is randomly selected from the group, find the probability that the number of hours spent by the student in doing homework is not more than the mode of the distribution. (2 marks)

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11. Let $f(x) = 2x^3 + ax^2 + bx - 6$. Given that $f(x)$ is divisible by $x + 1$. When $f(x)$ is divided by $x - 2$, the remainder is 12.

(a) Find the values of a and b. (4 marks)

(b) Someone claims that all roots of the equation $f(x) = 0$ are rational numbers. Do you agree? Explain your answer. (2 marks)

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12. Figure 3 shows a frustum with the height of 8 cm. The radii of the upper surface and lower surface are in the ratio of 3 : 5 and the volume of the frustum is 1176π cm^3.

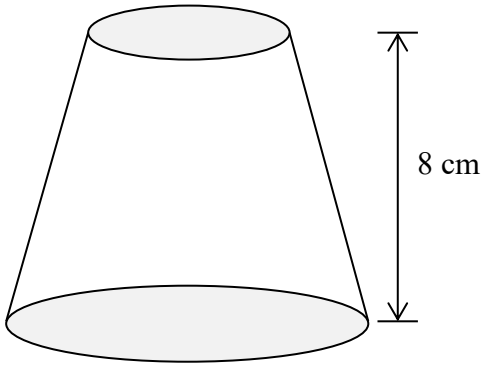

Figure 3

(a) Find the radius of the upper surface. (3 marks)

(b) Someone claims that the curved surface area of the frustum is larger than 250π cm^2. Do you agree? Explain your answer. (4 marks)

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13. The circle C passes through the point $A(2, 3)$ and $B(8, 11)$ and the centre of C is the point $G(9, 4)$.

(a) Find the equation of C. (2 marks)

(b) P is a moving point in the rectangular coordinate plane such that $AP = BP$. Denote the locus of P by Γ.

(i) Find the equation of Γ.

(ii) Describe the geometric relationship between Γ and $\angle AGB$.

(iii) If Γ cuts C at Q and R, find the area of the quadrilateral $AQBR$.

Leave your answer in surd form if necessary. (6 marks)

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14. It is given that f(x) is the sum of two parts, one part varies as x^2 and the other part varies as x. Suppose that f(–2) = 20 and f(17) = 153.

(a) Find f(2). (4 marks)

(b) $A(a, 9)$ and $B(b, 9)$ (where $b > a$) are points lying on the graph of y = f(x). Find the area of $\triangle ABC$ where C is a point lying on the x-axis. (4 marks)

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SECTION B (35 marks)

15. The mean of the test scores obtained by a class of students in a Chinese test is 55 marks. John gets 65 marks and his standard score is 0.5.

    (a) Find the standard deviation of the test scores. (1 mark)

    (b) Many students fail the test. The teacher decides to adjust the test scores. Extra 10 marks are added to each test score. Is there any change in the standard score of each student due to the score adjustment? Explain your answer. (2 marks)

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16. There are 8 white socks and 10 black socks in a drawer.

(a) If 2 socks are randomly drawn from the drawer at the same time, find the probability that 2 socks of the same colour are drawn. (2 marks)

(b) If 6 socks are randomly drawn from the drawer at the same time, find the probability that at least 4 black socks are drawn. (2 marks)

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17. Figure 4(a) shows a solid pyramid $VABCD$ with a rectangular base, where $AB = 20$ cm, $BC = 16$ cm, $VA = 24$ cm and $VB = VC = 32$ cm.

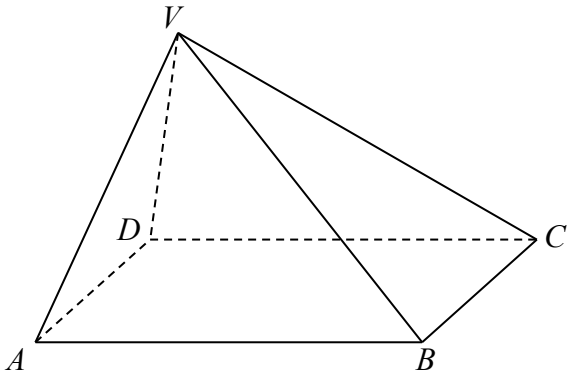

Figure 4(a)

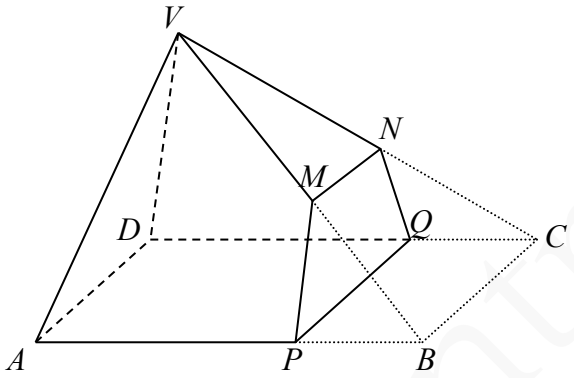

Figure 4(b)

(a) Find $\angle VBA$. (2 marks)

(b) M and N are the mid-points of VB and VC respectively. The plane $MNPQ$ is perpendicular to the plane $ABCD$. A geometric model is made by cutting off $PBCQNM$ from $VABCD$ as shown in Figure 4(b). Someone claims that the area of the trapezium $PQNM$ is less than 140 cm^2. Do you agree? Explain your answer. (5 marks)

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18. (a) In Figure 5, the equation of the straight ling L_1 is $4x + 11y = 1080$ and the y-intercept of the straight line L_2 is 140. L_1 and L_2 intersect at the point (50, 80). The shaded region (including the boundary) represents the solution of a system of inequalities. Find the system of inequalities. (4 marks)

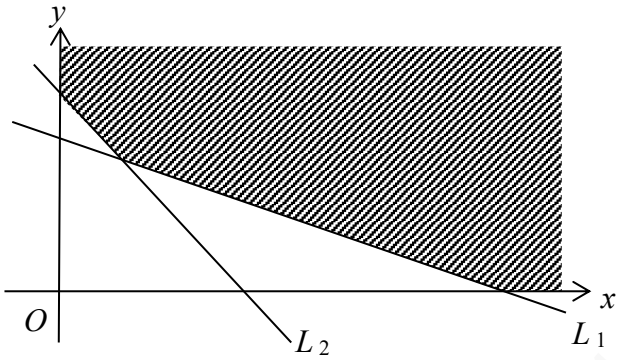

Figure 5

(b) A factory produces two types of T-shirts, X and Y. Each T-shirt X requires 4 units of polyester and 6 units of cotton for production while each T-shirt Y requires 11 units of polyester and 5 units of cotton for production. The supplier requires the factory to order at least 1080 units of polyester and 700 units of cotton. The costs for producing a T-shirt X and a T-shirt Y are \$30 and \$48 respectively. The manager claims that the total cost of producing the T-shirts always exceed \$5 000. Do you agree? Explain your answer. (4 marks)

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19. Peter buys a car worth $\$4 \times 10^5$ by yearly installment from a car company. The interest is at a fixed rate of r% per annum compounded yearly. He plans to pay an amount of $\$6 \times 10^4$ at the end of each year. At the start of the 3rd year, he owes the car company $\$3.58 \times 10^5$.

(a) (i) Express, in terms of r, the amount he owes the car company at the start of the 2nd year.

(ii) Find r.

(4 marks)

(b) (i) Express, in terms of n, the amount he owes the car company at the start of the $(n+1)$th year.

(ii) At the start of which year will the amount he owes the car company be less than $\$2 \times 10^5$?

(5 marks)

(c) At the start of the year he owes the car company an amount of less than $\$2 \times 10^5$, he buys another car. Using a similar payment method, he finds that the amount he owes the car company for the new car at the start of $(m+1)$th year since he buys the new car is $\$\left(a + b(1.21)^m\right)$. It is given that when $m = 1$, the amount he owes the car company for the new car is $\$1.2 \times 10^5$; when $m = 2$, the amount he owes the car company for the new car is $\$7.8 \times 10^4$. Peter claims that the amount he owes the car company for the 2nd car is always less than that for the 1st car. Is he correct? Explain your answer. (4 marks)

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