MOCK EXAM 4
MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

Name: _____

$(2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(m^4 n^{-2})^4}{(n^2)^{-3}}$ and express your answer with positive indices. (3 marks)

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2. Make a the subject of the formula $\dfrac{3+b}{5a-2} = -2b+1$. (3 marks)

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3. Factorize

   (a) $12x^2 + xy - 35y^2$,

   (b) $9x - 15y + 12x^2 + xy - 35y^2$.

   (3 marks)

4. Mr. Wong's monthly salary is 25% higher than that of his son, Peter while Peter's salary is 25% lower than that of Mrs. Wong. Who has the highest monthly salary? Explain your answer.

   (3 marks)

5.   (a)   Find the range of values of x which satisfy both $\dfrac{3x+5}{2} < 2(x+4)$ and $3x+14 \le 0$.

     (b)   How many negative integers satisfy both the inequalities in (a)?   (4 marks)

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6.   In a football league, each team gains 3 points for a win, 1 point for a draw and 0 point for a loss. The champion of the league plays 38 games and gains a total of 93 points. It is known that the champion loses three games. Find the number of games the champion wins.   (4 marks)

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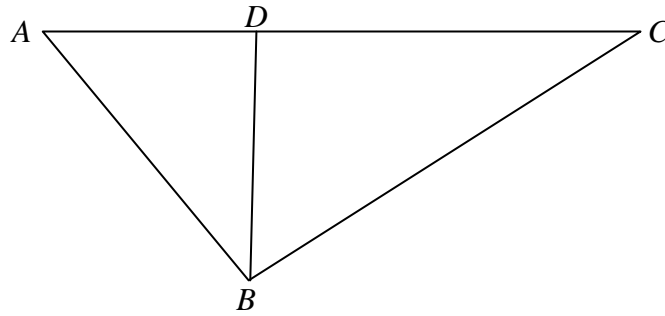7. In Figure 1, D is a point lying on AC such that $\angle ABD = \angle BCD$.

Figure 1

(a) Prove that $\triangle ABC \sim \triangle ADB$.

(b) Suppose that $AB = 15$ cm, $BC = 20$ cm and $AD = 9$ cm. Is $\triangle ABC$ a right-angled triangle? Explain your answer. (5 marks)

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8. The coordinates of the points A and B are (6, 4) and (3, 2) respectively. A is translated vertically downwards by 13 units to A'. B is rotated anti-clockwise about the origin O through 90° to B'.

   (a) Write down the coordinates of A' and B'.

   (b) Is AB perpendicular to $A'B'$? Explain your answer. (5 marks)

Answers written in the margins will not be marked.

9. A pack of rice is termed *supreme* if its weight is measured as 10.0 kg correct to the nearest 100 g.

   (a) Find the smallest possible weight of a *supreme* pack of rice.

   (b) Is it possible that the total weight of 88 *supreme* packs of rice is measured as 885 kg correct to the nearest kg ? Explain your answer. (5 marks)

Answers written in the margins will not be marked.

SECTION A(2) (35 marks)

10. It is given that g(x) is a partly constant and partly varies as $(x - 5)^2$. Suppose that g(2) = 10 and g(7) = 0.

    (a) Find g(0). (3 marks)

    (b) Denote the graph of y = g(x) – 10 by G.
        (i) Write down the y-intercept of G.
        (ii) Find the x-intercept(s) of G. (3 marks)

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11. There are 34 students in a class. The box-and-whisker diagram below shows the distribution of the weights of the students. It is given that the mean of the distribution is 44 kg.

(a) Find the range and the inter-quartile range of the above distribution. (3 marks)

(b) Four students of weights (in kg) 37.5, 41.2, 44.2 and 45.5 join the class. Find the mean and the median of the weights of the students in the class now. (3 marks)

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12. Let $f(x) = 2x^3 + ax^2 + bx - 24$, where a and b are constants. Given that $f(x)$ is divisible by $x - 4$. When $f(x)$ is divided by $x + 1$, the remainder is -5.

(a) Find the values of a and b. (4 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are real numbers. Do you agree? Explain your answer. (3 marks)

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13. Figure 2(a) shows a vessel in the form of a right circular cone half-filled with some water initially.
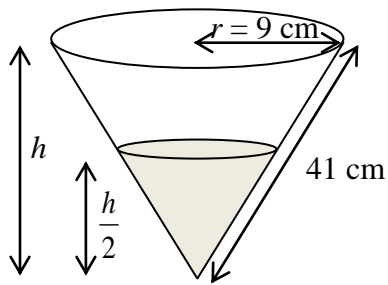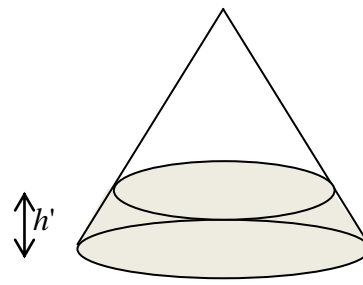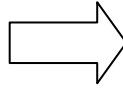

Figure 2(a)   Figure 2(b)

(a) Find the volume of the water in terms of π. (3 marks)

(b) The vessel is then flipped upside down to rest on a table as shown in Figure 2(b).

    (i) Find the new height, h'.

    (ii) Peter claims that the area of the wet curved surface of the vessel in Figure 2(b) is greater than that in Figure 2(a). Do you agree? Explain your answer. (5 marks)

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14. The *x*-intercept and *y*-intercept of *L* are 4 and 3 respectively. Denote the points by *A* and *B*.

(a) *C* is a circle passing through the origin, *A* and *B*. Find the equation of *C*. (3 marks)

(b) Let the coordinates of the point *D* be (2, –1). *P* is a moving point in the rectangular coordinate plane such that $AP = DP$. Denote the locus of *P* by *Γ*.

    (i) Find the equation of *Γ*.

    (ii) Describe the geometric relationship between *Γ* and the line segment *AD*.

    (iii) If *Γ* cuts *C* at *Q* and *R*, find the arc length of *QAR* in terms of *π*. (5 marks)

Answers written in the margins will not be marked.

SECTION B (35 marks)

15. The graph in Figure 3 shows the linear relation between $\log_9 x$ and $\log_{27} y$. The intercepts on the horizontal axis and the vertical axis of the graph are 2 and 1 respectively. Express the relation between x and y in the form $y = Ax^k$, where A and k are constants. (3 marks)

Figure 3

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16. There are 5 classes in S6 of a school. 3 representatives from each class are nominated to form an organizing committee to organize activities of the fun fair. 4 members are selected randomly from the organizing committee to take charge of financial issues.

    (a) Find the probability that the 4 selected members are nominated by 4 different classes.

    (2 marks)

    (b) Find the probability that the 4 selected members are nominated by at most 3 different classes.

    (2 marks)

17. The coordinates of the centre of the circle C are $(k, 6)$, where $k > 0$. It is given that the y-axis is a tangent to C.

(a) Find the equation of C in terms of k. (2 marks)

(b) The slope and the y-intercept of the straight line L is –2 and 2 respectively. If L cuts C at A and B, express the coordinates of the mid-point of AB in terms of k. (5 marks)

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18. Figure 4(a) shows a right pyramid *VABCD* with a square base, where $\angle VAB = 60^\circ$. The length of a side of the base is 30 cm. Let *P* and *Q* be the points lying on *VA* and *VD* respectively such that *PQ* is parallel to *BC* and *PA* = 20 cm. A geometric model is made by cutting off the pyramid *VPBCQ* from *VABCD* as shown in Figure 4(b).

Figure 4(a)　　　　　　　Figure 4(b)

(a) Find the length of *PB*. (2 marks)

(b) Find the angle between the plane *PBCQ* and the plane *PADQ*. (4 marks)

(c) A craftsman claims that the area of the trapezium *PBCQ* is less than 500 cm². Do you agree? Explain your answer. (2 marks)

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19. In a city, the number of patients treated by the hospital X is P(n) in the nth year since the start of its operation, where n is a positive integer. It is given that P(n) = ka^n, where k and a are positive constants. It is found that the numbers of patients treated by X in the 1st year and the 3rd year since the start of its operation are 13 800 and 19 872 respectively.

(a) (i) Find k and a.

   Hence find the number of patients treated by X in the 5th year since the start of its operation.

(ii) Express, in terms of n, the total number of patients treated by X in the first nth year since the start of its operation.

(6 marks)

(b) Hospital Y starts to operate since X has been operated for 5 years. Let Q(m) be the number of patients treated by the hospital Y in the mth year since the start of its operation, where m is a positive integer. It is given that $Q(m) = \dfrac{1}{2}ka^{2m}$.

(i) The hospital authority claims that after Y has been operated for 8 years, the number of patients treated by Y each year will exceed that of X. Do you agree? Explain your answer.

(ii) The hospital authority thinks that when the total number of patients treated by X and Y since the start of the operation of X exceeds 600 000, new facilities should be installed to maintain the efficiency of the hospitals. According to the hospital authority, in which year since the start of the operation of X should the new facilities be installed?

(7 marks)

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END OF PAPER

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