Suggested Solution for 2014 HKDSE Mathematics(core) Multiple Choice Questions

1. B

$(2n^3)^{-5}$

$=\frac{1}{(2n^3)^5}$

$=\frac{1}{2^5 n^{3\times 5}}$

$=\frac{1}{32n^{15}}$

2. A

$u^2 - v^2 - 5u + 5v$

$= (u + v)(u - v) - 5(u - v)$

$= (u - v)(u + v - 5)$

3. B

L.H.S. $= px(x - 1) + x^2$

$= px^2 - px + x^2$

$= (p + 1)x^2 - px$

R.H.S. $= qx(x - 2) + 4x$

$= qx^2 - 2qx + 4x$

$= qx^2 + (4 - 2q)x$

$$\begin{cases} p + 1 = q \ \ldots (1) \\ -p = 4 - 2q \ \ldots (2) \end{cases}$$

$(1) \times 2 + (2)$,

$p + 2 = 4$

$p = 2$

Alternatively

Substitute $x = 2$ into both sides,

$p(2)(2 - 1) + (2)^2 = q(2)(2 - 2) + 4(2)$

$2p + 4 = 8$

$p = 2$

4. B

$x^2 + ax + a = 1$

$x^2 + ax + a - 1 = 0$

$\triangle = 0$

$a^2 - 4(1)(a - 1) = 0$

$a^2 - 4a + 4 = 0$

$(a - 2)^2 = 0$

$a = 2$

5. C

∵ The graph opens upwards.

∴ $m > 0$

∵ The y-intercept < 0

∴ $n < 0$

6. D

For $0 > a > b$, $a^2 < b^2$.

∴ I may NOT be true.

For $a > b$, $a + k > b + k$

∴ II is true.

∵ $\frac{1}{k^2} > 0$ and $a > b$

∴ $\frac{a}{k^2} > \frac{b}{k^2}$

∴ III is true.

7. C

$-3x < 6$ and $6 < 2x$

$x > -2$ and $x > 3$

∴ $x > 3$

8. A

Let the price of a bowl and that of a cup be \$$x$ and \$$y$ respectively.

$$\begin{cases} 2x + 3y = 506 \;\ldots\; (1) \\ 5x = 4y \quad \text{i.e.} \quad 5x - 4y = 0 \;\ldots\; (2) \end{cases}$$

(1) × 4 + (2) × 3,

$23x = 2024$

$x = 88$

9. A

Let x be the number of female workers. Then, the number of male workers is $(1 - 20\%)x$ i.e. $0.8x$.

$x + 0.8x = 792$

$1.8x = 792$

$x = 440$

Number of male workers

$= 0.8(440)$

$= 352$

10. C

Let θ and r be the original angle and radius of the sector respectively.

$\pi[(1-50\%)r]^2 \times \frac{\theta(1-x\%)}{360°} = \pi r^2 \times \frac{\theta}{360°} \times (1-90\%)$

$0.25(1-x\%) = 0.1$

$x = 60$

11. A

The lower limit of $x = (8-0.5) \times (10-0.5) = 71.25$ cm^2

The upper limit of $x = (8+0.5) \times (10+0.5) = 89.25$ cm^2

$\therefore \quad 71.25 \leq x < 89.25$

12. D

Let $\frac{4}{5a} = \frac{5}{7b} = \frac{7}{9c} = k$ where k is a constant.

Then, $a = \frac{4}{5k}$, $b = \frac{5}{7k}$ and $c = \frac{7}{9k}$.

$\because \quad \frac{5}{7} < \frac{7}{9} < \frac{4}{5}$

$\therefore \quad b < c < a$

13. C

$z = \frac{ky^3}{x}$ where k is a constant.

$k = \frac{xz}{y^3}$

$\therefore \quad \frac{y^3}{xz} = \frac{1}{k}$ is a constant.

14. D

$a_4 = a_3 + a_2$

$63 = a_3 + 7$

$a_3 = 56$

$a_5 = a_4 + a_3$

$= 63 + 56$

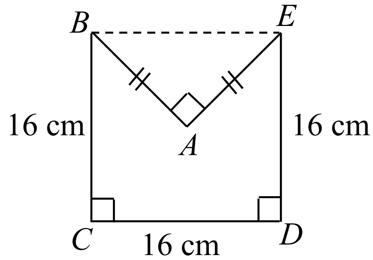
$= 119$

15. C

Note that $\triangle ABE$ is a right-angled $\triangle$ and $BCDE$ is a square.

The area of $\triangle ABE$ is one-fourth that of $BCDE$

$\therefore$ Area of $ABCDE = \frac{3}{4} \times 16 \times 16 = 192 \text{ cm}^2$



16. C

Note that $\triangle DAE \cong \triangle DCG$.

$\angle ADE = \angle CDG = 25^\circ$ (corr. $\angle$s, $\cong\triangle$s)

$\angle ADC = \angle ADE + \angle EDF + \angle CDF = 90^\circ$

$25^\circ + \angle EDF + 20^\circ = 90^\circ$

$\angle EDF = 45^\circ$

$\angle FDG = \angle CDF + \angle CDG = 20^\circ + 25^\circ = 45^\circ$

$\therefore \quad \angle EDF = \angle FDG = 45^\circ$

$DE = DG$ (corr. sides, $\cong\triangle$s)

$DF = DF$ (common)

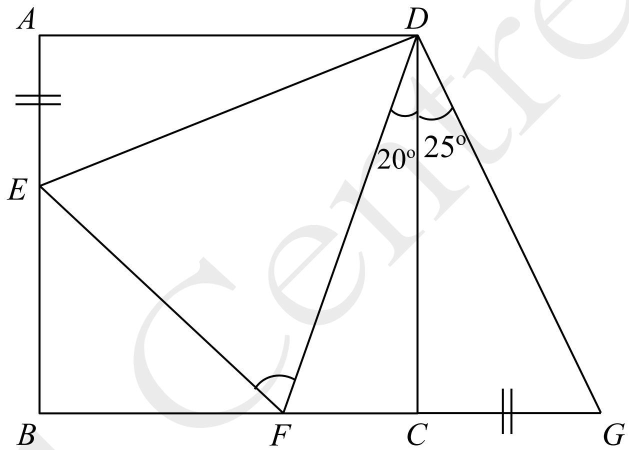
$\therefore \quad \triangle DEF \cong \triangle DGF$(SAS)

$\angle DFC + \angle CDF + \angle DCF = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle DFC + 20^\circ + 90^\circ = 180^\circ$

$\angle DFC = 70^\circ$

$\therefore \quad \angle DFE = \angle DFG = 70^\circ$ (corr. $\angle$s, $\cong\triangle$s)



17. D

Produce CD to meet AE at F.

Note that $\triangle CDB \sim \triangle CFA$.

$\therefore \quad FD : DC = AB : BC = 3 : 2$

Area of $\triangle EDF$: area of $\triangle EDC = FD : DC = 3 : 2$ ($\because$ $\triangle EDF$ and $\triangle EDC$ have the same height.)

Area of $\triangle EDF : 8 = 3 : 2$

Area of $\triangle EDF = 12 \text{ cm}^2$

$$\frac{\text{Area of } \Delta CFA}{\text{Area of } \Delta CDB} = \left(\frac{AC}{BC}\right)^2$$

$$\frac{\text{Area of } \Delta CDB + \text{area of } ABDF}{\text{Area of } \Delta CDB} = \left(\frac{3+2}{2}\right)^2$$

$$\frac{4 + \text{area of } ABDF}{4} = \left(\frac{5}{2}\right)^2$$

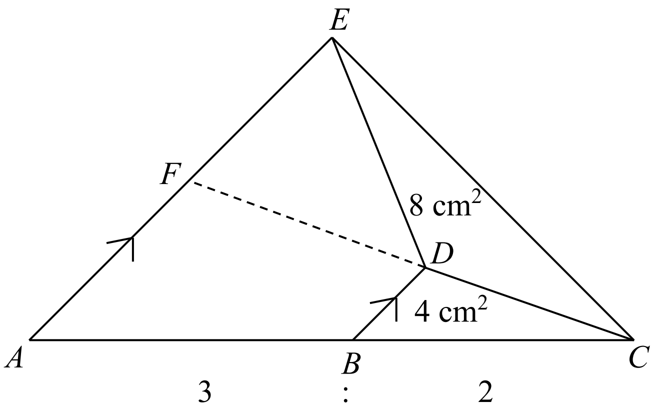
Area of $ABDF = 21 \text{ cm}^2$

Area of $ABDE$

= Area of $ABDF$ + area of $\triangle EDF$

= 21 + 12

= 33 cm^2



18. A

$\angle ADB = 90^\circ - \theta$

$\angle CDB = 90^\circ - (90^\circ - \theta) = \theta$

$\frac{DB}{AB} = \tan\theta$

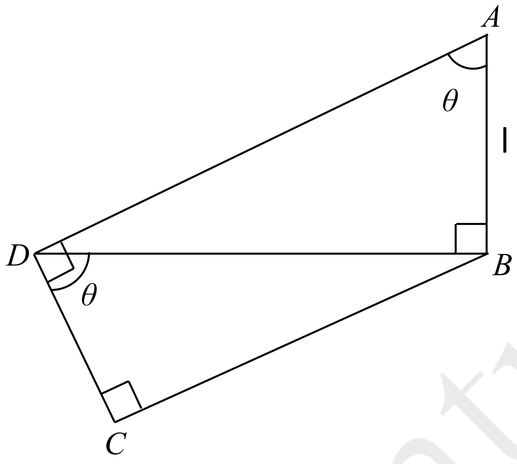
$DB = AB \tan\theta = l \tan\theta$

$\frac{CD}{DB} = \cos\theta$

$CD = DB \cos\theta$

$= l \tan\theta \cos\theta$

$= l \sin\theta$



19. A

$(\cos(90° + \theta) + 1)(\sin(360° - \theta) - 1)$

$= (-\sin\theta + 1)(-\sin\theta - 1)$

$= (-\sin\theta)^2 - 1$

$= \sin^2\theta - 1$

$= -\cos^2\theta$

20. B

Join AB.

$\angle ABE = \angle ADE$ ($\angle$s in the same segment)

$= 28^\circ$

$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)

$\angle CBE = \angle ABC - \angle ABE$

$= 90^\circ - 28^\circ$

$= 62^\circ$

Alternatively

Join CD.

$\angle ADC = 90^\circ$ ($\angle$ in semi-circle)

$\angle CDE = \angle ADC + \angle ADE$

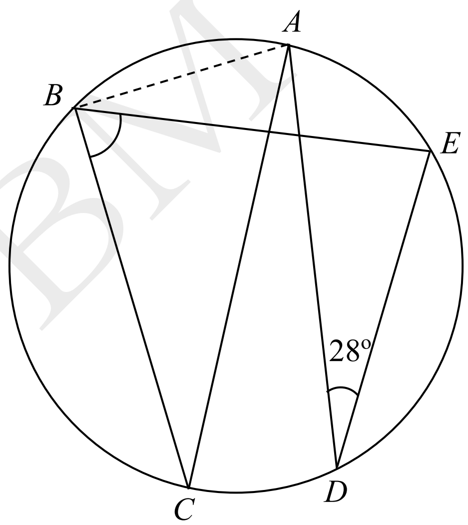
$= 90^\circ + 28^\circ$

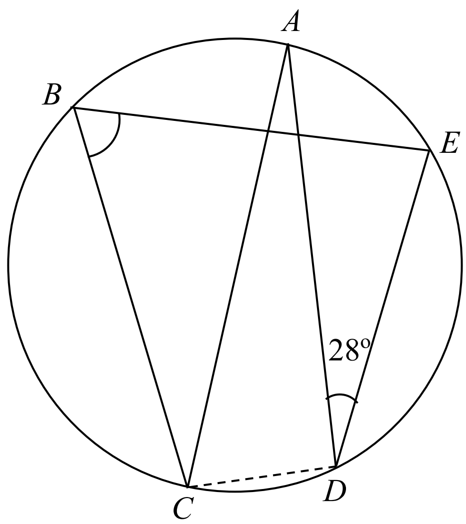
$= 118^\circ$

$\angle CBE + \angle CDE = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle CBE + 118^\circ = 180^\circ$

$\angle CBE = 62^\circ$

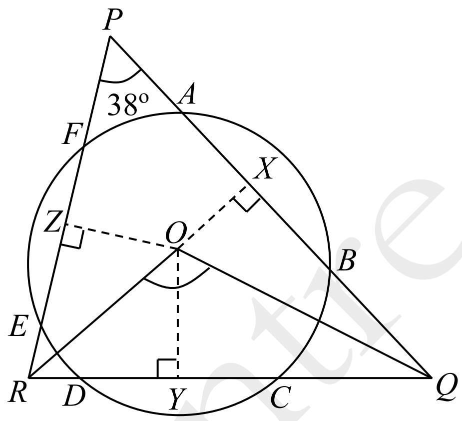




21. A

Let X, Y and Z be the feet of the perpendiculars from O to AB, CD and EF respectively.

Then, $OX = OY = OZ$. (equal chords, equidistant from centre)



Note that $\triangle OXQ \cong \triangle OYQ$ and $\triangle OYR \cong \triangle OZR$. (RHS)

Let $\angle ORY = \angle ORZ = x$ and $\angle OQX = \angle OQY = y$.

$\angle PRQ + \angle PQR + \angle QPR = 180^\circ$ ($\angle$ sum of $\triangle$)

$2x + 2y + 38^\circ = 180^\circ$

$x + y = 71^\circ$

$\angle QOR + \angle ORQ + \angle OQR = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle QOR + x + y = 180^\circ$

$\angle QOR + 71^\circ = 180^\circ$

$\angle QOR = 109^\circ$

22. C

Let x be an exterior angle of the regular polygon. Then, an interior angle is $180^\circ - x$.

$(180^\circ - x) - x = 100^\circ$

$x = 40^\circ$

$n = \dfrac{360}{40} = 9$

$\therefore$ I is NOT true and II is true.

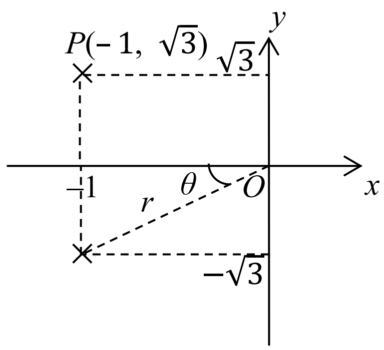
$\because$ $n = 9$

$\therefore$ The number of axes of reflectional symmetry of the polygon is 9.

$\therefore$ III is true.

23. B

After reflection, the coordinates of the image of P are $(-1, -\sqrt{3})$.



$r = \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$= 2$

$\tan\theta = \dfrac{\sqrt{3}}{1}$

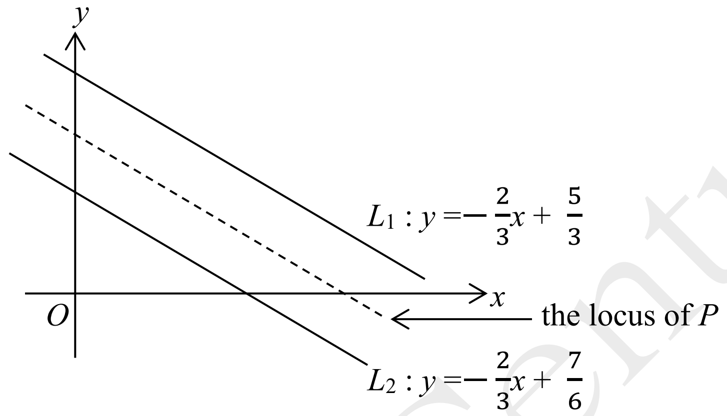
$\theta = 60^\circ$

$\therefore$ The polar coordinates of the image are $(2, 180^\circ + 60^\circ)$. i.e. $(2, 240^\circ)$

24. D

Rewrite $L_1 : y = -\frac{2}{3}x + \frac{5}{3}$ and $L_2 : y = -\frac{2}{3}x + \frac{7}{6}$. Note that $L_1 // L_2$.

As shown below, the locus of P is a straight line parallel to both L_1 and L_2 and is equidistant from L_1 and L_2.



25. D

Rewrite the equations of the straight lines as $y = -\frac{a}{b}x + \frac{1}{b}$ and $y = -\frac{c}{d}x + \frac{1}{d}$.

From the figure, $\frac{1}{b} > 0$ and $-\frac{a}{b} > 0$.

∴ $b > 0$ and $a < 0$.

∴ I is true.

From the figure, $\frac{1}{d} > 0$ and $-\frac{c}{d} < 0$.

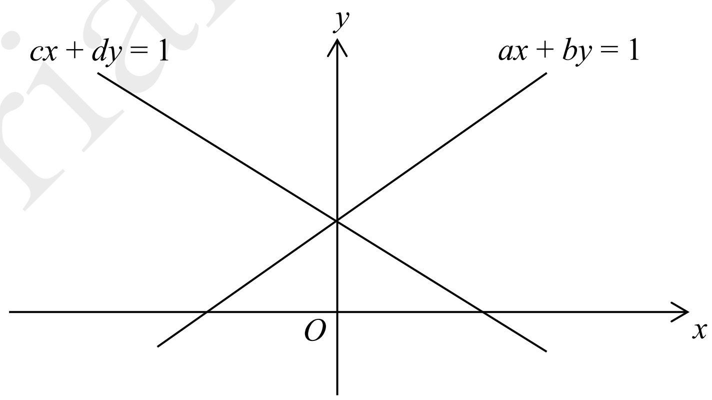
∴ $d > 0$ and $c > 0$.

∴ II is true.

$\frac{1}{b} = \frac{1}{d}$

∴ $b = d$

∴ III is true.



26. A

Centre $= (-\frac{-8}{2}, -\frac{k}{2})$ i.e. $(4, -\frac{k}{2})$

Then, $\frac{-\frac{k}{2}-(-5)}{4-6} = -4$

$k = -6$

27. B

$\frac{m}{m+20}=\frac{1}{m}$

$m^2 - m - 20 = 0$

$(m - 5)(m + 4) = 0$

$m = 5$ or -4 (rejected)

28. D

Let m be the mean height of the teachers.

$\frac{25m+145\times140}{25+140} = 150$ cm

$m = 178$ cm

29. C

Let x be his expenditure on transportation that week.

x : \$240 = (360° – 160° – 50° – 90°) : 160°

x : \$240 = 60° : 160°

x = \$90

30. B

∵ $0 \le h \le 4$

∴ I is NOT true.

Note that $0 \le k \le 9$ … (1)

∵ The range $\ge$ 33

∴ $40 + k - (10 + h) \ge 33$

$k - h \ge 3$ i.e. $k \ge 3 + h$ … (2)

Combining (1) and (2), $3 \le k \le 9$

∴ II must be true.

The largest possible value of $k - h = 9 - 0 = 9$

∴ $3 \le k - h \le 9$

∴ III is NOT true.

31. A

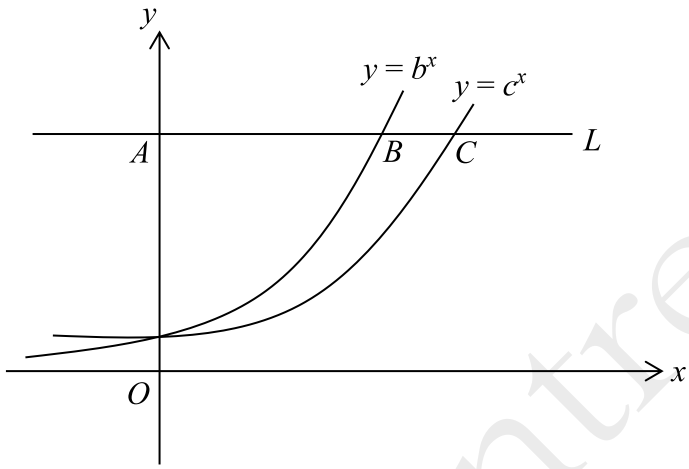
The H.C.F. of 3, 4 and 6 is 1.

The H.C.F. of x^4, x and x^2 is x.

The H.C.F. of y^2, y^5 and y^3 is y^2.

∴ The required H.C.F. is xy^2.

32. C



For the same value of $x > 0$,

$b^x > c^x$

∴ $b > c$

∴ I is NOT true.

Note that $b > 1$ and $c > 1$.

∴ $bc > 1$

∴ II is true.

$b^{AB} = c^{AC}$

$\log b^{AB} = \log c^{AC}$

$(AB)\log b = (AC)\log c$

$\frac{AB}{AC} = \frac{\log c}{\log b} = \log_b c$

∴ III is true.

33. B

$\log 124^{241} = 241 \log 124 \approx 505$

$\log 241^{214} = 214 \log 241 \approx 510$

$\log 412^{142} = 142 \log 412 \approx 371$

$\log 421^{124} = 124 \log 421 \approx 325$

∴ 241^{214} is the greatest.

34. C

$7 \times 2^{10} + 2^8 + 5 \times 2^3 - 2^3$

$= (2^2 + 2 + 1) \times 2^{10} + 2^8 + 4 \times 2^3$

$= 2^{12} + 2^{11} + 2^{10} + 2^8 + 2^2 \times 2^3$

$= 2^{12} + 2^{11} + 2^{10} + 2^8 + 2^5$

$= 1110100100000_2$

2^{12}	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	0	1	0	0	1	0	0	0	0	0

35. D

$f(x) = 3x^2 - 6x + k$

$= 3(x^2 - 2x + 1) + k - 3$

$= 3(x - 1)^2 + k - 3$

∴ $k - 3 = 7$

$k = 10$

Alternatively

The axis of symmetry is $x = -\frac{(-6)}{2(3)}$ i.e. $x = 1$

Then, $f(1) = 7$.

$3(1)^2 - 6(1) + k = 7$

$k = 10$

36. A

$\frac{\beta^2+4}{\beta+2i}$

$=\frac{\beta^2+4}{\beta+2i}\times\frac{\beta-2i}{\beta-2i}$

$=\frac{(\beta^2+4)(\beta-2i)}{\beta^2-(2i)^2}$

$=\frac{(\beta^2+4)(\beta-2i)}{\beta^2+4}$

$=\beta-2i$

37. B

$\frac{2^{2m}}{2^m}=\frac{2^{3m}}{2^{2m}}=\frac{2^{4m}}{2^{3m}}=2^m$

∴ I is a geometric sequence.

$\frac{2m^2}{m}=2m$ but $\frac{3m^4}{2m^2}=\frac{3m^2}{2}$

∴ II is NOT a geometric sequence.

$\frac{\log m^2}{\log m}=\frac{\log m^4}{\log m^2}=\frac{\log m^8}{\log m^4}=2$

∴ III is a geometric sequence.

38. A

$y=1-f(x)$ implies a reflection about the x-axis and a translation of 1 unit upwards.

Note that:-

B shows a reflection about the x-axis and a translation of 1 unit to the left.

C shows a reflection about the x-axis and a translation of 1 unit downwards.

D shows a reflection about the x-axis and a translation of 1 unit to the right.

39. D

$7\sin^2 x=\sin x$

$7\sin^2 x-\sin x=0$

$\sin x(7\sin x-1)=0$

$\sin x=0$ or $\sin x=\frac{1}{7}$

$x=0^\circ, 8.21^\circ, 172^\circ, 180^\circ$ or 360°

40. D

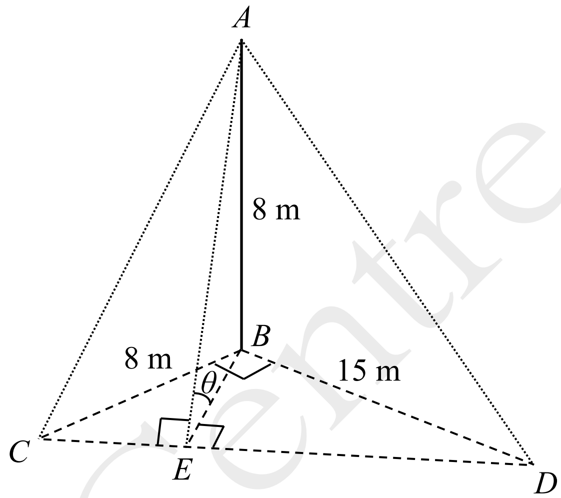
Let E be the foot of the perpendicular from A to CD. Then, $\theta = \angle AEB$.

By Pythagoras' theorem,

$CD^2 = BC^2 + BD^2$

$= 8^2 + 15^2$

$CD = 17$ m



Area of $\Delta BCD = \frac{1}{2} \times BC \times BD = \frac{1}{2} \times BE \times CD$

$\therefore \frac{1}{2} \times 8 \times 15 = \frac{1}{2} \times BE \times 17$

$BE = \frac{120}{17}$ m

$\tan\theta = \frac{AB}{BE} = \frac{8}{\frac{120}{17}}$

$= \frac{17}{15}$

41. C

Note that $\angle IRS = \angle IRQ = 12^\circ$ ($\because$ I is the in-centre of ΔQRS)

$\angle SRQ = \angle IRS + \angle IRQ$

$= 12^\circ + 12^\circ$

$= 24^\circ$

$\angle QSR = \angle QPS$ ($\angle$s in alt. segment)

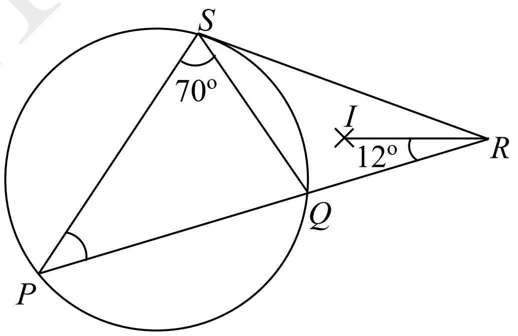
$\angle PSR = \angle PSQ + \angle QSR$

$= 70^\circ + \angle QPS$

$\angle QPS + \angle PSR + \angle SRQ = 180^\circ$ ($\angle$ sum of Δ)

$\angle QPS + (70^\circ + \angle QPS) + 24^\circ = 180^\circ$

$\angle QPS = 43^\circ$



42. C

$\begin{cases} x - y = k \text{ i.e. } y = x - k \ldots (1) \\ x^2 + y^2 + 2x - 4y - 1 = 0 \ldots (2) \end{cases}$

Substitute (1) into (2),

$x^2 + (x - k)^2 + 2x - 4(x - k) - 1 = 0$

$x^2 + x^2 - 2kx + k^2 + 2x - 4x + 4k - 1 = 0$

$2x^2 - 2(1 + k)x + k^2 + 4k - 1 = 0$

The x-coordinate of the mid-point of AB

$= \frac{1}{2} \cdot \frac{2(1+k)}{2}$

$= \frac{1+k}{2}$

43. B

Number of teams that can be formed

$= C_2^{13} C_3^{17}$

= 53 040

44. D

Let μ and σ be the mean and the standard deviation of the examination scores respectively.

$\frac{55-\mu}{\sigma} = -3$ i.e. $\mu - 3\sigma = 55$ … (1)

$\frac{95-\mu}{\sigma} = 2$ i.e. $\mu + 2\sigma = 95$ … (2)

(1) × 2 + (2) × 3,

$5\mu = 395$

$\mu = 79$

45. B

Multiplying each number by –1 will change the variance by a factor of $(-1)^2$ i.e. 1. Adding 14 to each number has no effect on the variance. Hence, the variance remains unchanged.

<u>Alternatively</u>

Let μ be the mean of the numbers. Then,

$\mu = \frac{a+b+c+d}{4}$

and the variance, $\sigma^2 = \frac{(a-\mu)^2+(b-\mu)^2+(c-\mu)^2+(d-\mu)^2}{4}$

The mean of $14-a$, $14-b$, $14-c$ and $14-d$ is $\frac{14-a+14-b+14-c+14-d}{4}$ i.e $14-\mu$.

The variance of $14-a$, $14-b$, $14-c$ and $14-d$ is

$\frac{[14-a-(14-u)]^2+[14-b-(14-u)]^2+[14-c-(14-u)]^2+[14-d-(14-u)]^2}{4}$

$= \sigma^2$