

Suggested Solution for 2015 HKDSE Mathematics(core) Multiple Choice Questions

1. D

$$\begin{aligned}
 & (x+1)(x^2+x+1) \\
 &= x(x^2+x+1) + 1 \cdot (x^2+x+1) \\
 &= x^3+x^2+x+x^2+x+1 \\
 &= x^3+2x^2+2x+1
 \end{aligned}$$

2. D

$$\begin{aligned}
 & \frac{(3y^6)^4}{3y^2} \\
 &= \frac{3^4 y^{6 \times 4}}{3y^2} \\
 &= 3^{4-1} y^{24-2} \\
 &= 27y^{22}
 \end{aligned}$$

3. A

$$\begin{cases} p + 3q = 4 \dots (1) \\ 5p + 9q = 2 \dots (2) \end{cases}$$

$$\begin{aligned}
 & (2) - (1) \times 3, \\
 & 2p = -10 \\
 & p = -5
 \end{aligned}$$

Alternatively

$$\begin{aligned}
 & \text{From (1), } p = 4 - 3q \dots (3) \\
 & \text{Substitute (3) into (2),} \\
 & 5(4 - 3q) + 9q = 2 \\
 & 20 - 15q + 9q = 2 \\
 & 6q = 18 \\
 & q = 3 \\
 & \text{Then, } p = 4 - 3(3) \\
 & \quad = -5
 \end{aligned}$$

4. D

$$\begin{aligned}
 & 0.0023456789 \\
 &= 0.002346 \text{ (correct to 6 decimal places)} \\
 &= 0.00234568 \text{ (correct to 6 significant figures)}
 \end{aligned}$$

5. B

$$\begin{aligned}
 & \text{R.H.S.} \\
 &= (x+4)(x-m) + 6 \\
 &= x^2 + (4-m)x + 6 - 4m \\
 & \text{By comparing the coefficients of } x, \\
 & m = 4 - m \\
 & m = 2 \\
 & \text{By comparing the constant terms,} \\
 & n = 6 - 4(2) = -2
 \end{aligned}$$

Alternatively

$$\begin{aligned}
 & \text{Substitute } x = 0, \\
 & n = 6 - 4m \dots (1) \\
 & \text{Substitute } x = -4, \\
 & (-4)^2 + m(-4) + n = 6 \\
 & 4m = n + 10 \dots (2) \\
 & \text{Substitute (2) into (1),} \\
 & n = 6 - (n + 10) \rightarrow n = -2
 \end{aligned}$$

6. A

$$18 + 7x > 4 \text{ or } 5 - 2x < 3$$

$$7x > -14 \text{ or } 2x > 2$$

$$x > -2 \text{ or } x > 1$$

$$\therefore x > -2$$

7. A

$$\therefore \beta \text{ is a root of the equation } 4x^2 - 5x - 1 = 0$$

$$\therefore 4\beta^2 - 5\beta - 1 = 0$$

$$8\beta^2 - 10\beta - 2 = 0$$

$$10\beta - 8\beta^2 = -2$$

$$7 + 10\beta - 8\beta^2 = 7 - 2 = 5$$

8. D

\therefore The graph opens upwards.

$$\therefore a > 0$$

\therefore The axis of symmetry is $x = -b < 0$

$$\therefore b > 0$$

9. B

Let x be the original price of the souvenir.

$$\begin{aligned} \text{New price of the souvenir, } x' &= (1 + 70\%)(1 - 60\%)x \\ &= 0.68x \end{aligned}$$

$$\begin{aligned} \text{The percentage change} &= \frac{0.68x - x}{x} \times 100\% \\ &= -32\% \end{aligned}$$

10. D

The required amount

$$= \$50\,000 \times \left(1 + \frac{6\%}{4}\right)^{3 \times 4}$$

$$= \$59\,781 \text{ (correct to the nearest dollar)}$$

11. C

$$a : c = 5 : 3 = 10 : 6$$

$$b : c = 3 : 2 = 9 : 6$$

$$\therefore a : b : c = 10 : 9 : 6$$

Let $a = 10k$, $b = 9k$ and $c = 6k$ where k is a constant.

$$(a + c) : (b + c)$$

$$= (10k + 6k) : (9k + 6k)$$

$$= 16 : 15$$

12. D

Let $z = kx^3y^2$ where k is a constant.

$$14 = k(2)^3(1)^2$$

$$k = \frac{7}{4}$$

Then, when $x = 3$ and $y = -2$,

$$\begin{aligned} z &= \frac{7}{4}(3)^3(-2)^2 \\ &= 189 \end{aligned}$$

Alternatively

$$14 = k(2)^3(1)^2 \dots (1)$$

$$z = k(3)^3(-2)^2 \dots (2)$$

$$(2) \div (1),$$

$$\frac{z}{14} = \frac{27}{2}$$

$$z = 189$$

13. B

Let $T(n)$ be the number of dots in the n th pattern. Then,

$$T(1) = 5$$

$$T(2) = 5 + 4 = 9$$

$$T(3) = 9 + 4 = 13$$

$$T(4) = 13 + 4 = 17$$

$$T(5) = 17 + 4 = 21$$

$$T(6) = 21 + 4 = 25$$

14. C

Greatest possible value of n

$$< \frac{(5+0.5) \times 1000}{10-0.5}$$

$$\approx 578.9473684$$

15. C

By Pythagoras' theorem,

$$CN^2 + DN^2 = CD^2$$

$$CN^2 + 6^2 = 10^2$$

$$CN = 8 \text{ cm}$$

By Pythagoras' theorem,

$$AN^2 + EN^2 = AE^2$$

$$AN^2 + 5^2 = 13^2$$

$$AN = 12 \text{ cm}$$

$$AC = 8 + 12 = 20 \text{ cm}$$

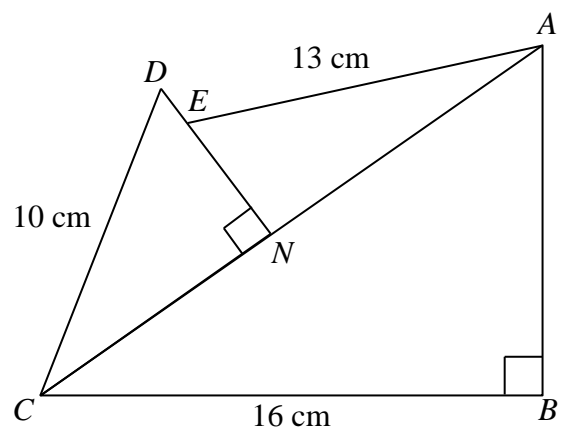
By Pythagoras' theorem,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 16^2 = 20^2$$

$$AB = 12 \text{ cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$$



16. B

Let r cm be the radius of the upper surface.

$$\frac{r}{9} = \frac{4}{12}$$

$$r = 3$$

Volume of the frustum

$$= \frac{1}{3}\pi(9)^2(12) - \frac{1}{3}\pi(3)^2(4)$$

$$= 312\pi \text{ cm}^3$$

Alternatively,

Volume of the circular cone

$$= \frac{1}{3}\pi(9)^2(12)$$

$$= 324\pi \text{ cm}^3$$

Let $V \text{ cm}^3$ be the volume of the upper cone being cut off.

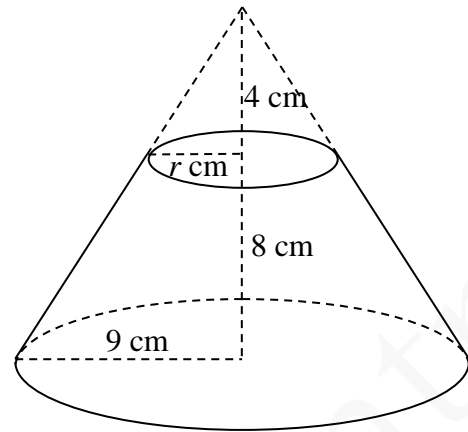
$$\frac{V}{324\pi} = \left(\frac{4}{12}\right)^3 \quad [\text{For similar figures}]$$

$$V = 12\pi \text{ cm}^3$$

Volume of the frustum

$$= 324\pi - 12\pi$$

$$= 312\pi \text{ cm}^3$$



17. C

Note that $\triangle EDF \sim \triangle ECB$.

$$\therefore DF : CB = DE : EC = 2 : 3$$

$$\therefore AD = BC \text{ (property of // gram)}$$

$$\therefore DF : AD = 2 : 3$$

Area of $\triangle ADE$: area of $\triangle DEF = AD : DF = 3 : 2$ ($\because \triangle ADE$ and $\triangle DEF$ have the same height.)

$$\text{Area of } \triangle ADE : 8 = 3 : 2$$

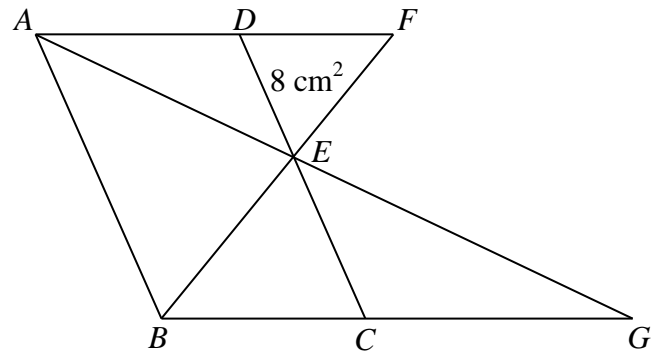
$$\text{Area of } \triangle ADE = 12 \text{ cm}^2$$

Note also that $\triangle DEA \sim \triangle CEG$.

$$\therefore \frac{\text{Area of } \triangle CEG}{\text{Area of } \triangle DEA} = \left(\frac{CE}{DE}\right)^2$$

$$\text{i.e. } \frac{\text{Area of } \triangle CEG}{12} = \left(\frac{3}{2}\right)^2$$

$$\therefore \text{Area of } \triangle CEG = 27 \text{ cm}^2$$



18. A

$$\frac{AC}{AB} = \tan \beta$$

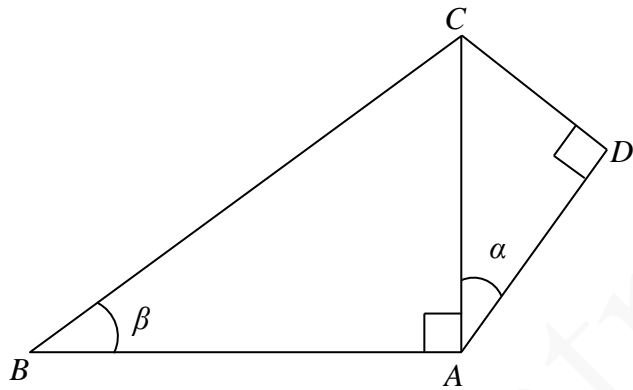
$$AC = AB \tan \beta \dots (*)$$

$$\frac{AD}{AC} = \cos \alpha$$

$$AD = AC \cos \alpha$$

$$= AB \tan \beta \cos \alpha \text{ [From (*)]}$$

$$\therefore \frac{AD}{AB} = \cos \alpha \tan \beta$$



19. C

$$\frac{\cos 180^\circ}{1 + \sin(90^\circ + \theta)} + \frac{\cos 360^\circ}{1 + \sin(270^\circ + \theta)}$$

$$= \frac{-1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta}$$

$$= \frac{-(1 - \cos \theta) + 1 + \cos \theta}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{-1 + \cos \theta + 1 + \cos \theta}{1 - \cos^2 \theta}$$

$$= \frac{2 \cos \theta}{\sin^2 \theta}$$

20. C

Join AC.

$$\angle ACD = 90^\circ \text{ (}\angle \text{ in semi-circle)}$$

$$\widehat{BC} = \widehat{CD} \text{ (equal chords, equal arcs)}$$

$$\angle BAC = \angle CAD \text{ (arcs prop. to } \angle \text{s at } \odot^{\text{cc}})$$

$$\therefore \angle CAD = \angle BAC = \frac{58^\circ}{2} = 29^\circ$$

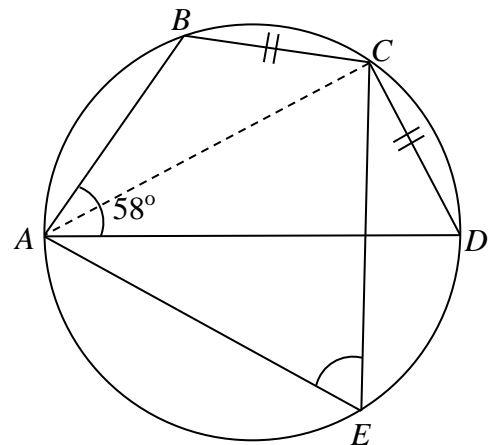
$$\angle CDA + \angle CAD + \angle ACD = 180^\circ \text{ (}\angle \text{ sum of } \triangle)$$

$$\angle CDA + 29^\circ + 90^\circ = 180^\circ$$

$$\angle CDA = 61^\circ$$

$$\angle AEC = \angle CDA \text{ (}\angle \text{s in the same segment)}$$

$$= 61^\circ$$



21. B

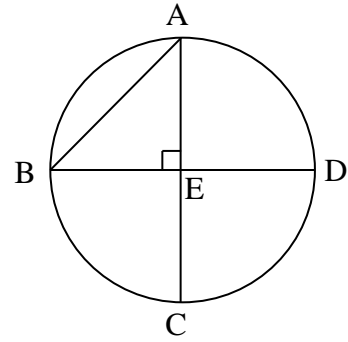
Note that E is the centre of the circle.

Then, the radius = $AE = BE = 12$ cm.

Area of $\triangle AEB$

$$= \frac{1}{2} \times 12 \times 12$$

$$= 72 \text{ cm}^2$$



22. A

Let x be an exterior angle of the regular polygon.

$$5x + x = 180^\circ$$

$$x = 30^\circ$$

Then, each interior angle = $5(30^\circ) = 150^\circ$

\therefore I is true.

$$\text{Number of sides} = \frac{360^\circ}{30^\circ} = 12 \text{ (sum of ext. } \angle\text{s of polygon)}$$

$$\text{Number of diagonals} = \frac{12(12-3)}{2} = 54$$

\therefore II is NOT true.

Number of folds of rotational symmetry is 12.

\therefore III is NOT true.

23. C

After reflection, the coordinates of the image of A are $(-\sqrt{3}, -1)$.

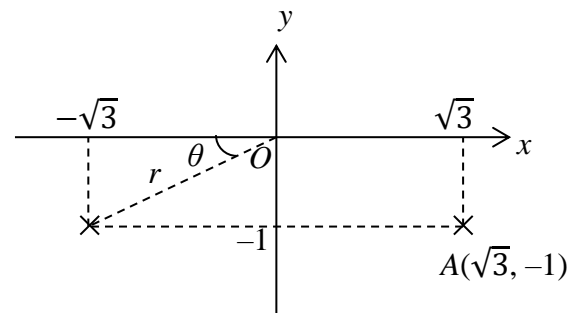
$$r = \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$= 2$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

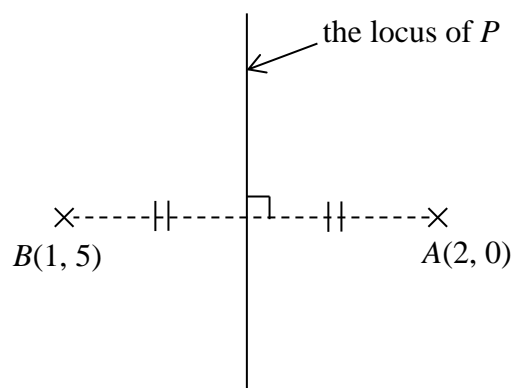
$$\theta = 30^\circ$$

\therefore The polar coordinates of the image are $(2, 180^\circ + 30^\circ)$. i.e. $(2, 210^\circ)$



24. A

As shown below, the locus of P is the perpendicular bisector of AB .



25. B

For L_1 , $x = \frac{1}{a} < 0$

$\therefore a < 0$

\therefore I is true.

For L_2 , when $y = 0$, $bx = 1$. i.e. $x = \frac{1}{b} < 0$

$\therefore b < 0$

Also, from the figure, $\frac{1}{b} > \frac{1}{a}$.

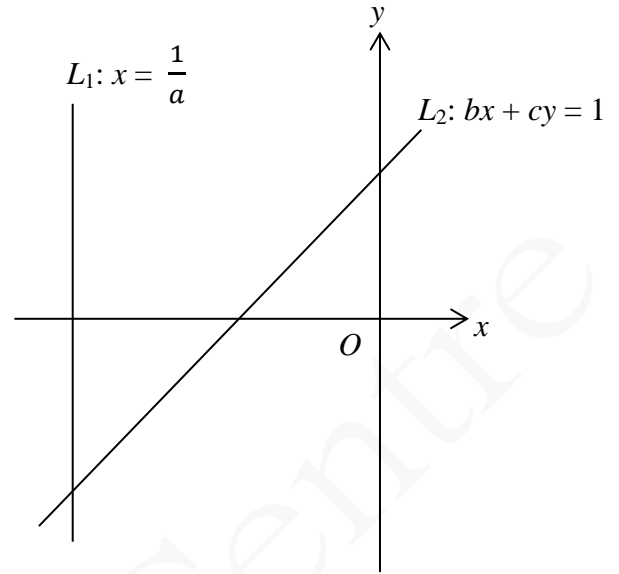
$\therefore a > b$

\therefore II is NOT true.

For L_2 , when $x = 0$, $cy = 1$. i.e. $y = \frac{1}{c} > 0$

$\therefore c > 0$

\therefore III is true.



26. C

Radius = 4

The equation of C is

$$[x - (-4)]^2 + (y - 3)^2 = 4^2$$

i.e. $x^2 + y^2 + 8x - 6y + 9 = 0$.

27. A

Refer to the table. There are 6 outcomes giving the sum of the numbers as 7.

Expected gain

$$= \frac{6}{36} \times \$36 + \frac{30}{36} \times \$6$$

= \$11

The number on 1st die

The number on 2nd die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

28. B

The required probability

$$= \frac{8}{12+16+8+4+4}$$

= $\frac{2}{11}$

29. A

The inter-quartile range

= 45 - 25

= 20

30. B

Note that $r = 3$.

$$\therefore 3 \leq m \leq 5$$

$$\therefore 3 \leq m = q \leq 5$$

$$\frac{2 \times 2 + 3 \times 6 + 5 \times 2 + 6 + 8 \times 2 + 9 + 10}{15} \leq p \leq \frac{2 \times 2 + 3 \times 5 + 5 \times 3 + 6 + 8 \times 2 + 9 + 10}{15}$$

$$\text{i.e. } \frac{73}{15} \leq p \leq 5$$

When $q = 5$, I is NOT true.

$$r(=3) < \frac{73}{15} \leq p \leq 5$$

 \therefore II must be true.
When $q = 3$, III is NOT true.

31. C

$$\begin{aligned} & \frac{1}{x^2 - 2x + 1} - \frac{1}{x^2 + x - 2} \\ &= \frac{1}{(x-1)^2} - \frac{1}{(x-1)(x+2)} \\ &= \frac{x+2-(x-1)}{(x-1)^2(x+2)} \\ &= \frac{3}{(x-1)^2(x+2)} \end{aligned}$$

32. A

Slope of the graph = $\frac{2-0}{0-3} = -\frac{2}{3}$ and $\log_3 y$ -intercept of the graph = 2

Equation of the straight line is

$$\log_3 y = -\frac{2}{3} \log_3 x + 2 \quad [\text{c.f. } y = mx + c]$$

$$\log_3 y = \log_3 x^{-\frac{2}{3}} + \log_3 3^2 \quad [\text{using } a \log b = \log b^a]$$

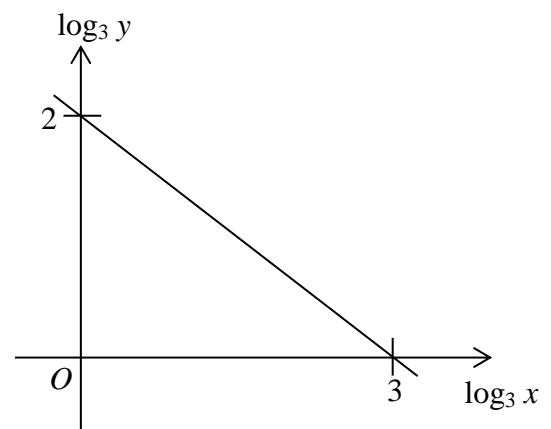
$$\log_3 y = \log_3 (9x^{-\frac{2}{3}}) \quad [\text{using } \log a + \log b = \log(ab)]$$

$$y = 9x^{-\frac{2}{3}}$$

$$x^{\frac{2}{3}}y = 9$$

$$(x^{\frac{2}{3}}y)^3 = 9^3$$

$$x^2y^3 = 729$$



33. A

$$\begin{aligned}
 11 &= 2^3 + 2^1 + 2^0 \\
 &= 1011_2 \\
 11 + 2^6 + 2^{10} + 2^{11} \\
 &= 110001001011_2
 \end{aligned}$$

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	0	0	1	0	0	1	0	1	1

34. B

$$\begin{aligned}
 \alpha + \beta &= -k \text{ and } \alpha\beta = -2 \\
 \alpha^2 + \beta^2 \\
 &= (\alpha + \beta)^2 - 2\alpha\beta \\
 &= (-k)^2 - 2(-2) \\
 &= k^2 + 4
 \end{aligned}$$

35. C

$$\begin{aligned}
 z &= (a + 5)i^6 + (a - 3)i^7 \\
 &= (a + 5)i^2 + (a - 3)i^3 \quad [\because i^4 = 1] \\
 &= (a + 5)(-1) + (a - 3)(-i) \\
 &= -a - 5 - (a - 3)i
 \end{aligned}$$

If z is a real number,

$$\begin{aligned}
 a - 3 &= 0 \\
 a &= 3
 \end{aligned}$$

36. D

The system of inequalities that define the shaded region is

$$\begin{cases}
 x \leq 4 \dots (1) \\
 y \leq x + 5 \dots (2) \\
 x + 2y \geq 10 \dots (3)
 \end{cases}$$

From (1), $a \leq 4$. \therefore I is true.From (2), $b \leq a + 5$ i.e. $a \geq b - 5$ \therefore II is true.From (3), $a + 2b \geq 10$ i.e. $a \geq 10 - 2b$ \therefore III is true.

37. D

Let r be the common ratio of the geometric sequence.

$$r^2 = \frac{x_8}{x_6} = \frac{96}{216} = \frac{4}{9}$$

$$r = \frac{2}{3} \text{ or } -\frac{2}{3}$$

For $r = \frac{2}{3}$,

$$x_3 = \frac{x_6}{r^3} = \frac{216}{\left(\frac{2}{3}\right)^3} = 729$$

For $r = -\frac{2}{3}$,

$$x_3 = \frac{x_6}{r^3} = \frac{216}{\left(-\frac{2}{3}\right)^3} = -729$$

\therefore I may NOT be true.

$$\frac{x_5}{x_7} = \frac{1}{r^2} = \frac{9}{4} > 1$$

\therefore II is true.

$$x_2 = \frac{x_6}{r^4} = \frac{216}{\left(\frac{4}{9}\right)^2} = \frac{2187}{2}$$

Note that x_{2n} are positive numbers for $n = 1, 2, 3, \dots$

$$x_2 + x_4 + x_6 + \dots + x_{2n}$$

$$< x_2 + x_4 + x_6 + \dots$$

$$= \frac{2187}{2}$$

$$= 1093.5$$

$$< 2015$$

\therefore III is true.

38. B

$$\cos^2 x - \sin x = 1$$

$$(1 - \sin^2 x) - \sin x = 1$$

$$\sin^2 x + \sin x = 0$$

$$\sin x(\sin x + 1) = 0$$

$$\sin x = 0 \text{ or } \sin x = -1$$

$$x = 0^\circ, 180^\circ \text{ or } 270^\circ$$

\therefore There are 3 roots.

39. D

From the figure,

$$\sin[k(75)^\circ + \theta] = 0 \text{ and } \sin[k(165)^\circ + \theta] = 0.$$

$$\text{i.e. } k(75)^\circ + \theta = 180^\circ \dots (1)$$

$$\text{and } k(165)^\circ + \theta = 360^\circ \dots (2)$$

Solving (1) and (2),

$$k = 2 \text{ and } \theta = 30^\circ$$

AlternativelyFrom the figure, the period of the graph = 180° .

\therefore It is a reduction of the graph $y = \sin x^\circ$ to 2 times along the x -axis. i.e. $k = 2$

The graph shows a shift of 15° to the left. Then,

$$y = \sin[2(x + 15)^\circ]$$

$$= \sin(2x + 30^\circ) \quad \text{i.e. } \theta = 30^\circ$$

40. C

Let G be the centre of the circle. Then, G is the mid-point of BD and GB , GC and GD are the radii.

$$\angle GBA = \angle GCA = 90^\circ \quad (\text{tangent} \perp \text{radius})$$

$$AC = AB \quad (\text{tangent properties})$$

$$= 6 \text{ cm}$$

$$CE = AE - AC$$

$$= 10 - 6$$

$$= 4 \text{ cm}$$

By Pythagoras' theorem,

$$AB^2 + BE^2 = AE^2$$

$$6^2 + BE^2 = 10^2$$

$$BE = 8 \text{ cm}$$

Let r cm be the radius of the circle.

$$\text{Then, } GE = (8 - r) \text{ cm}$$

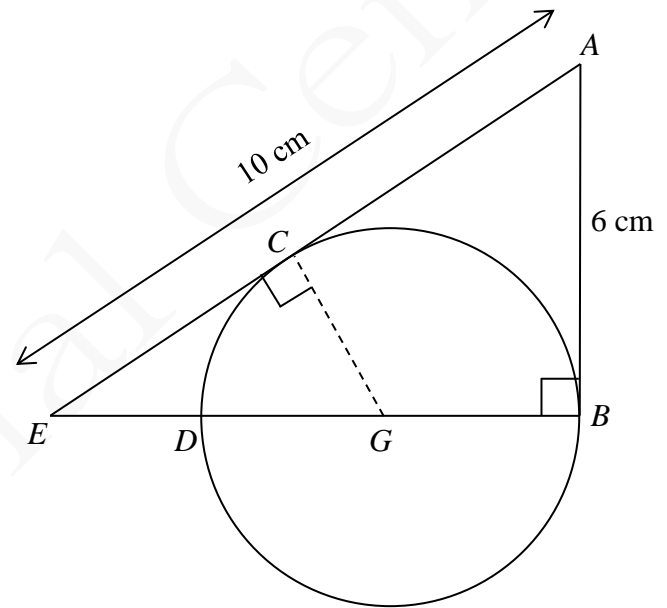
By Pythagoras' theorem,

$$GC^2 + CE^2 = GE^2$$

$$r^2 + 4^2 = (8 - r)^2$$

$$r = 3 \text{ cm}$$

$$BD = 2r = 6 \text{ cm}$$



41. B

$$\begin{cases} x + y + 4 = 0 & \text{i.e. } x = -y - 4 \dots (1) \\ x^2 + y^2 + 2x - 6y + k = 0 \dots (2) \end{cases}$$

$$\begin{cases} x^2 + y^2 + 2x - 6y + k = 0 \dots (2) \end{cases}$$

Substitute (1) into (2),

$$(-y - 4)^2 + y^2 + 2(-y - 4) - 6y + k = 0$$

$$y^2 + 8y + 16 + y^2 - 2y - 8 - 6y + k = 0$$

$$2y^2 + 8 + k = 0$$

$$\Delta = 0$$

$$\therefore \Delta = 0^2 - 4(2)(8 + k) = 0$$

$$\therefore k = -8$$

42. A

Let T be the foot of the perpendicular from O to PQ .

Slope of $OT \times$ slope of $PQ = -1$

$$\text{Slope of } OT \times \frac{60-48}{0-96} = -1$$

Slope of $OT = 8$

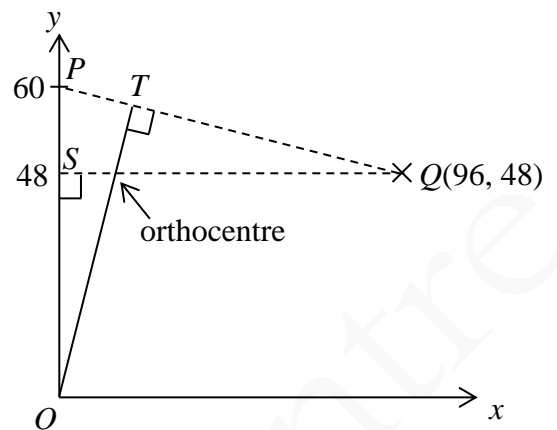
\therefore The equation of OT is $y = 8x$.

The orthocentre is the intersection of QS and OT .

Note that the y -coordinate of the orthocentre is 48.

Then, $48 = 8x$

$$x = 6$$



43. C

The girls can occupy the indicated positions (marked with \uparrow).

Number of queues that can be formed

$$= P_6^6 \times P_2^7$$

$$= 30\,240$$

Alternatively

Consider the two girls are next to each other. Take them as a group that forms a queue with the 6 boys.

Number of queues that can be formed

$$= P_7^7 \times P_2^2$$

$$= 100\,800$$

Number of queues required

$$= P_8^8 - 100\,800$$

$$= 30\,240$$

$\uparrow B_1 \uparrow B_2 \uparrow B_3 \uparrow B_4 \uparrow B_5 \uparrow B_6 \uparrow$

44. D

The required probability

$= P(\text{"choosing bag } P") \times P(\text{"choosing a green ball"}) + P(\text{"choosing bag } Q") \times P(\text{"choosing a green ball"})$

$$= \frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{3}{4}$$

$$= \frac{17}{24}$$

45. B

$$x_1 = \frac{a_1 + a_2 + a_3 + \dots + a_{49} + a_{50}}{50} \quad \text{i.e.} \quad a_1 + a_2 + a_3 + \dots + a_{49} + a_{50} = 50x_1 \dots (1)$$

$$x_2 = \frac{a_1 + a_2 + a_3 + \dots + a_{48} + a_{49}}{49} \quad \text{i.e.} \quad a_1 + a_2 + a_3 + \dots + a_{48} + a_{49} = 49x_2 \dots (2)$$

Substitute (2) into (1),

$$49x_2 + a_{50} = 50x_1$$

$$49x_2 + x_1 = 50x_1 \quad [\because x_1 = a_{50}]$$

$$x_1 = x_2$$

\therefore I is true.

Without loss of generality, let $a_1 \leq a_2 \leq a_3 \leq \dots \leq a_{49}$.

Then, $y_2 = a_{25}$.

If $a_{50} \geq a_{25}$, then $y_1 \geq y_2$.

If $a_{50} \leq a_{25}$, then $y_1 \leq y_2$.

\therefore II may NOT be true.

Since $a_{50} = x_1$, the group of numbers $\{a_1, a_2, a_3, \dots, a_{50}\}$ is less dispersed than the group of numbers $\{a_1, a_2, a_3, \dots, a_{49}\}$.

$$\therefore z_1 \leq z_2$$

\therefore III is true.

Alternatively,

$$\begin{aligned} z_1 &= \frac{(a_1 - x_1)^2 + (a_2 - x_1)^2 + \dots + (a_{49} - x_1)^2 + (a_{50} - x_1)^2}{50} \\ &= \frac{(a_1 - x_1)^2 + (a_2 - x_1)^2 + \dots + (a_{49} - x_1)^2}{50} \quad [\because x_1 = a_{50}] \end{aligned}$$

$$\begin{aligned} z_2 &= \frac{(a_1 - x_2)^2 + (a_2 - x_2)^2 + \dots + (a_{49} - x_2)^2}{49} \\ &= \frac{(a_1 - x_1)^2 + (a_2 - x_1)^2 + \dots + (a_{49} - x_1)^2}{49} \quad [\because x_1 = x_2] \end{aligned}$$

$$\geq z_1$$

\therefore III is true.