Suggested Solution for 2016 HKDSE Mathematics(core) Multiple Choice Questions

1. A

$8^{222} \bullet 5^{666}$
$= (2^3)^{222} \bullet 5^{666}$
$= 2^{666} \bullet 5^{666}$
$= (2 \bullet 5)^{666}$
$= 10^{666}$

2. A

$\frac{a}{x} + \frac{b}{y} = 3$

$\frac{a}{x} = 3 - \frac{b}{y}$

$= \frac{3y-b}{y}$

$\frac{x}{a} = \frac{y}{3y-b}$

$x = \frac{ay}{3y-b}$

Alternatively

$\frac{a}{x} + \frac{b}{y} = 3$

$ay + bx = 3xy$

$3xy - bx = ay$

$x(3y - b) = ay$

$x = \frac{ay}{3y-b}$

3. D

$16 - (2x - 3y)^2$
$= [4 - (2x - 3y)][4 + (2x - 3y)]$
$= (4 - 2x + 3y)(4 + 2x - 3y)$

4. C

0.0765403
= 0.077 (correct to 2 significant figures)
= 0.077 (correct to 3 decimal places)
= 0.07654 (correct to 4 significant figures)
= 0.07654 (correct to 5 decimal places)

5. A

$\begin{cases} 4\alpha + \beta = 5 \ \ldots (1) \\ 7\alpha + 3\beta = 5 \ \ldots (2) \end{cases}$

(2) × 4 – (1) × 7,
$5\beta = -15$
$\beta = -3$

Alternatively
From (1), $\beta = 5 - 4\alpha \ \ldots (3)$
Substitute (3) into (2),
$7\alpha + 3(5 - 4\alpha) = 5$
$\alpha = 2 \ \ldots (4)$
Substitute (4) into (3),
$\beta = 5 - 4(2)$
$= -3$

6. B

By factor theorem, $f(-\frac{1}{2}) = 0$.

$4(-\frac{1}{2})^3 + k(-\frac{1}{2}) + 3 = 0$

$k = 5$

The remainder required

$= f(-1)$

$= 4(-1)^3 + 5(-1) + 3$

$= -6$

7. A

$-5x > 21 - 2x$ and $6x - 18 < 0$

$-3x > 21$ and $6x < 18$

$x < -7$ or $x < 3$

$\therefore \quad x < -7$

8. C

$\Delta = 0$

$k^2 - 4(1)(8k + 36) = 0$

$k^2 - 32k - 144 = 0$

$(k + 4)(k - 36) = 0$

$k = -4$ or 36

9. D

$y = (ax + 1)^2 + a$

$y = a^2x^2 + 2ax + 1 + a$

$\because \quad a^2 > 0$

$\therefore$ The graph opens upwards.

$\because \quad -1 < a < 0$

$\therefore \quad a + 1 > 0$ i.e. the y-intercept > 0

10. C

The monthly salary of Peter

= \$33 360 ÷ (1 + 25%)

= \$26 688

The monthly salary of Teresa

= \$26 688 ÷ (1 – 25%)

= \$35 584

11. D

$\frac{3y-4x}{2x+y}=\frac{5}{6}$

$6(3y - 4x) = 5(2x + y)$

$18y - 24x = 10x + 5y$

$34x = 13y$

$\frac{x}{y}=\frac{13}{34}$ i.e. $x : y = 13 : 34$

12. D

Let $z = \frac{k\sqrt{x}}{y}$.

New value of z, $z' = \frac{k\sqrt{(1-36\%)x}}{(1+60\%)y}$

$= \frac{0.5k\sqrt{x}}{y}$

$= 0.5z$

% change in z

$= \frac{0.5z-z}{z} \times 100\%$

$= -50\%$

13. A

Let $\$y$/kg be the cost of flour of brand Y. Then,

$\frac{\$42\times3+\$y\times2}{3+2} = \$36$

$126 + 2y = 180$

$2y = 54$

$y = 27$

14. C

Let T(n) be the number of dots in the nth pattern. Then,

T(1) = 9

T(2) = 9 + 5 = 14

T(3) = 14 + 5 = 19

T(4) = 19 + 5 = 24

T(5) = 24 + 5 = 29

T(6) = 29 + 5 = 34

T(7) = 34 + 5 = 39

15. B

Draw a straight line parallel to the lines as shown.

$x + c = 180^o$ (int. $\angle$s, // lines)

$x = 180^o - c$

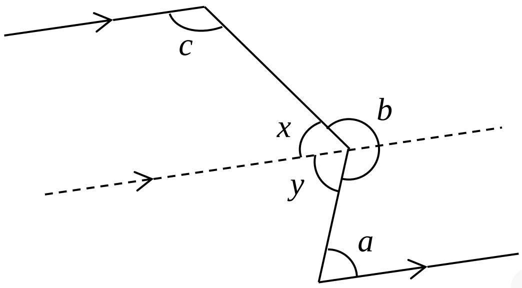
$y = a$ (alt. $\angle$s, // lines)

$x + y + b = 360^o$ ($\angle$s at a pt.)

$(180^o - c) + a + b = 360^o$

$a + b - c = 180^o$

$\therefore$ II must be true.



16. D

$AB^2 + BD^2 = 24^2 + 32^2$

$= 40^2$

$= AD^2$

$\therefore$ $\triangle ABD$ is a right-angled $\triangle$ with $\angle ABD = 90^o$

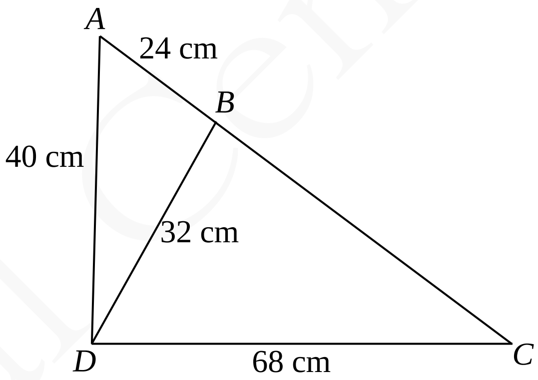
Then, $\angle CBD = 90^o$.

By Pythagoras' theorem,

$BD^2 + BC^2 = CD^2$

$32^2 + BC^2 = 68^2$

$BC = 60$ cm



17. A

$\angle ECB + \angle ADC = 180^o$ (int. $\angle$s, AD // BC)

$\angle ECB + 114^o = 180^o$

$\angle ECB = 66^o$

$\angle EBC = \angle ECB$ (base $\angle$s, isos. $\triangle$)

$= 66^o$

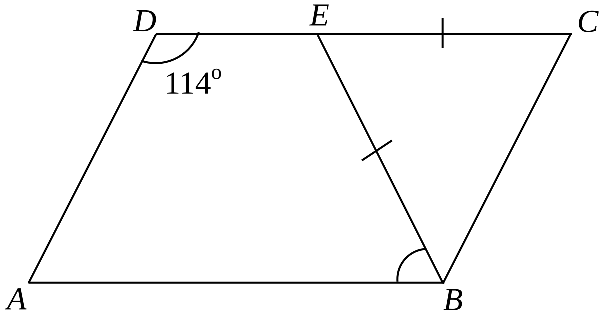
$\angle ABC = \angle ADC$ (opp. $\angle$s of // gram)

$= 114^o$

$\therefore$ $\angle ABE = \angle ABC - \angle EBC$

$= 114^o - 66^o$

$= 48^o$



18. C

By Pythagoras' Theorem,

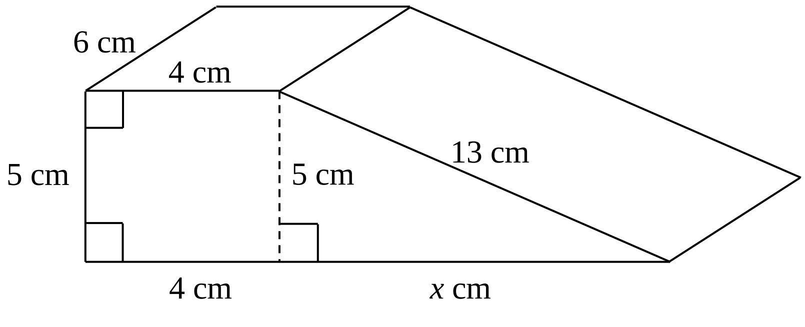
$x^2 + 5^2 = 13^2$

$x = 12$

Volume of the prism

$= \dfrac{(4+4+12)\times 5}{2} \times 6$

$= 300$ cm^3



19. A

Area of the shaded region = $\pi(39^2 - 33^2) \times \frac{\angle AOB}{360^o} = 72\pi$

$\angle AOB = 60^o$

$\therefore$ I is true.

Area of the sector *OAB*

$= \pi(33)^2 \times \frac{60^o}{360^o}$

$= 118.5\pi$ cm^2

$\therefore$ II is not true.

$\overset{\frown}{CD}$

$= 2\pi(39) \times \frac{60^o}{360^o}$

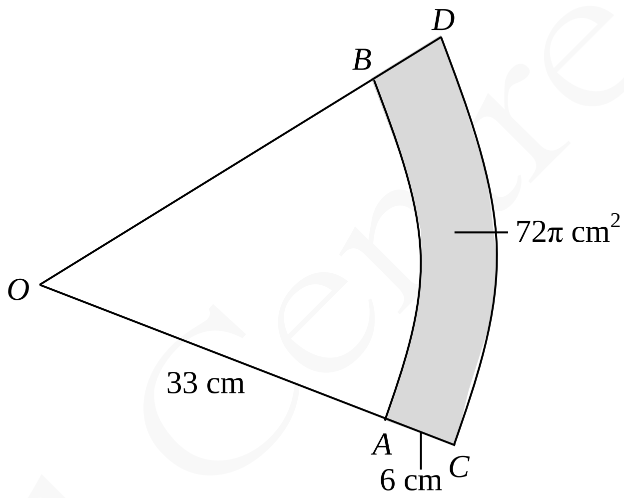
$= 13\pi$ cm

Perimeter of the sector *OCD*

$= 13\pi + 39 + 39$

$= (13\pi + 78)$ cm

$\therefore$ III is not true.



20. C

Note that area of quadrilateral *GHEQ* = area of quadrilateral *ABCP*.

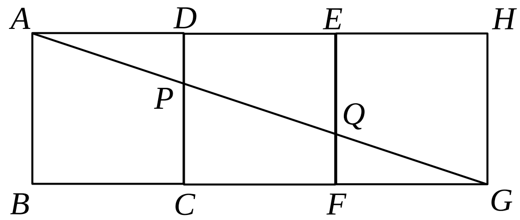
Also note that $\triangle ADP \sim \triangle AEQ \sim \triangle AHG$ and $AD : AE : AH = 1 : 2 : 3$.

$\therefore \frac{Area\ of\ \Delta ADP}{Area\ of\ \Delta AEQ} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and $\frac{Area\ of\ \Delta ADP}{Area\ of\ \Delta AHG} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\therefore \frac{Area\ of\ quadrilateral\ DEQP}{Area\ of\ quadrilateral\ GHEQ}$

$= \frac{4-1}{9-4}$

$= \frac{3}{5}$



i.e. Area of quadrilateral *DEQP* : area of quadrilateral *ABCP* = 3 : 5

21. B

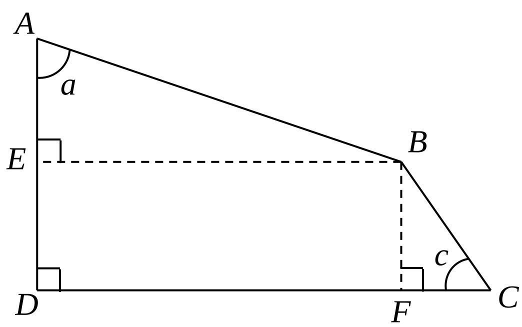
Construct *BE* and *BF* such that *BE* // *CD* and *BF* // *AD* as shown. Then,

$AE = AB \cos a$ and $BF = BC \sin c$

AD

$= AE + BF$

$= AB \cos a + BC \sin c$



22. D

$\angle BCD + \angle ADC = 180^\circ$ (int. $\angle$s, $BC//AD$)

$\angle BCD + 118^\circ = 180^\circ$

$\angle BCD = 62^\circ$

$\angle BED = \frac{1}{2}\angle BCD$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

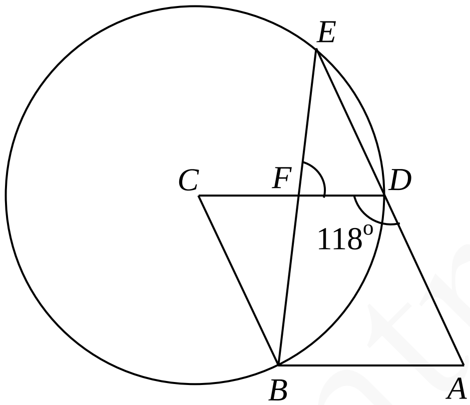
$= \frac{1}{2} \times 62^\circ$

$= 31^\circ$

$\angle FED + \angle DFE = \angle ADC$ (ext. $\angle$ of $\triangle$)

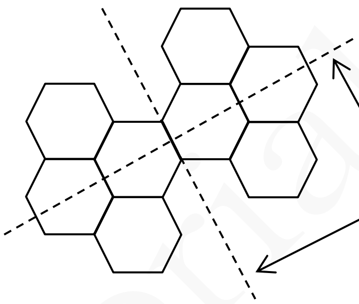
$31^\circ + \angle DFE = 118^\circ$

$\angle DFE = 87^\circ$



23. A

As shown, there are 2 axes of reflectional symmetry.



24. B

$(n - 2) \times 180^\circ = 3240^\circ$

$n = 20$

Each exterior angle

$= 360^\circ \div 20$

$= 18^\circ$

25. D

For $4x + 3y - 5 = 0$, when $y = 0$, $4x - 5 = 0$. i.e. $x = \frac{5}{4}$

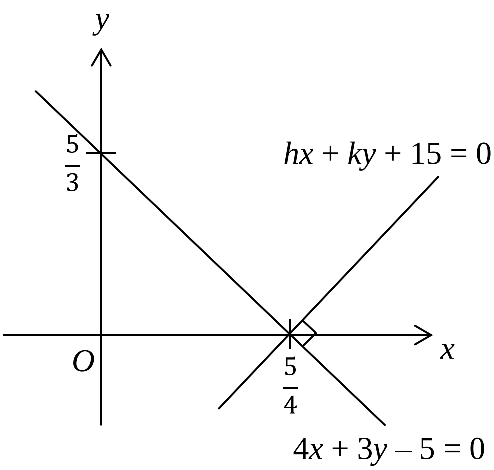
For $hx + ky + 15 = 0$, when $y = 0$, $hx + 15 = 0$. i.e. $x = -\frac{15}{h}$

$\therefore \quad -\frac{15}{h} = \frac{5}{4}$ i.e. $h = -12$

Rewrite the equations as $y = \frac{12}{k}x - \frac{15}{k}$ and $y = -\frac{4}{3}x + \frac{5}{3}$. Then,

$\left(\frac{12}{k}\right) \times \left(-\frac{4}{3}\right) = -1$

$\therefore \quad k = 16$



26. B

Let the y-coordinate of C be k.

∵ C lies on $x - 2y = 0$.

∴ The x-coordinate of C is $2k$. Then, by distance formula,

$$\sqrt{(9-2k)^2+(-2-k)^2} = \sqrt{(-1-2k)^2+(8-k)^2} \quad [\because AC = BC]$$

$81 - 36k + 4k^2 + 4 + 4k + k^2 = 1 + 4k + 4k^2 + 64 - 16k + k^2$

$k = 1$

∴ The x-coordinate of C is 2.

27. C

Rewrite the equation of the circle C as $x^2 + y^2 - 4x + 10y + \frac{65}{3} = 0$.

The centre of $C = \left(-\frac{(-4)}{2}, -\frac{10}{2}\right)$

$= (2, -5)$

∴ III is true.

The radius of C, $= \sqrt{(2)^2+(-5)^2-\frac{65}{3}}$

$= \sqrt{\frac{22}{3}} \neq 14$

∴ I is NOT true.

The distance between the centre and the origin

$= \sqrt{(2)^2+(-5)^2}$

$= \sqrt{29}$

$> \sqrt{\frac{22}{3}}$

∴ II is true.

28. C

The favourable outcomes are ($1, $2, $5), ($1, $2, $10), ($1, $5, $10) and ($2, $5, $10).

3 of the outcomes are at least $13.

∴ The required probability = $\frac{3}{4}$

29. B

The expected number

$= \frac{1}{10} \times 90 + \frac{3}{10} \times 20 + \frac{6}{10} \times 10$

$= 21$

30. B

∵ Mode = 68

∴ There must be two "68"s in a, b and c.

Let's take $a = b = 68$. Then,

$$\frac{32+68\times3+79+86+88+98\times2+c}{10} = 77$$

$c = 85$

∴ The median $= \frac{79+85}{2}$

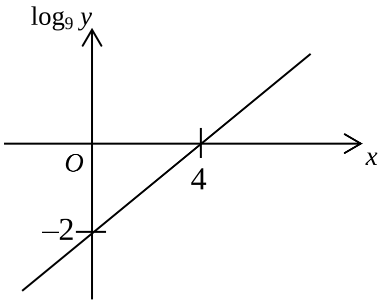
$= 82$

31. C

The L.C.M. of 9, 12 and 15 is 180.

∴ The L.C.M. required is $180a^6b^3$.

32. D



Slope of the graph $= \frac{-2-0}{0-4} = \frac{1}{2}$ and $\log_9 y$–intercept of the graph = –2

Equation of the straight line is

$\log_9 y = \frac{1}{2}x - 2$

$y = 9^{\frac{1}{2}x-2}$

$= 9^{-2} \bullet \left(9^{\frac{1}{2}}\right)^x$

$= \frac{1}{81} \bullet 3^x$

∴ $a = \frac{1}{81}$ and $b = 3$

<u>Alternatively</u>

$y = ab^x$

$\log_9 y = \log_9(ab^x)$

$= \log_9 a + \log_9 b^x$ [∵ $\log_9(xy) = \log_9 x + \log_9 y$]

$= (\log_9 b)x + \log_9 a$ [∵ $\log_9 x^y = y\log_9 x$]

∴ Slope $= \log_9 b = \frac{1}{2}$ i.e. $b = 9^{\frac{1}{2}} = 3$

33. A

$BC000DE000000_{16}$

$= 11 \times 16^{12} + 12 \times 16^{11} + 13 \times 16^7 + 14 \times 16^6$

$= (11 \times 16 + 12) \times 16^{11} + (13 \times 16 + 14) \times 16^6$

$= 188 \times 16^{11} + 222 \times 16^6$

16^{12}	16^{11}	16^{10}	16^9	16^8	16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0
B	C	0	0	0	D	E	0	0	0	0	0	0

34. B

uv

$= \frac{7}{a+i} \times \frac{7}{a-i}$

$= \frac{49}{a^2+1}$

$\because$ a can be an irrational number e.g. π

$\therefore$ I may not be true.

$u = \frac{7}{a+i}$

$= \frac{7}{a+i} \times \frac{a-i}{a-i}$

$= \frac{7a}{a^2+1} - \frac{7}{a^2+1}i$

$v = \frac{7}{a-i}$

$= \frac{7}{a-i} \times \frac{a+i}{a+i}$

$= \frac{7a}{a^2+1} + \frac{7}{a^2+1}i$

The real part of u = the real part of v = $\frac{7a}{a^2+1}$

$\therefore$ II is true.

$\frac{1}{u}$

$= \frac{a+i}{7}$

$= \frac{a}{7} + \frac{1}{7}i$

$\frac{1}{v}$

$= \frac{a-i}{7}$

$= \frac{a}{7} - \frac{1}{7}i$

The imaginary part of $\frac{1}{u}$ is $\frac{1}{7}$ and the imaginary part of $\frac{1}{v}$ is $-\frac{1}{7}$.

$\therefore$ III is not true.

35. D

$7y - 5x + 3$ attains its greatest value if y is the greatest and x is the smallest.

$\because$ S has the greatest y-coordinate and the smallest x-coordinate among P, Q, R and S

$\therefore$ S is the required point.

36. B

Let r be the common ratio of the geometric sequence.

$a_1r^2 = 21 \ldots (1)$

$a_1r^6 = 189 \ldots (2)$

Substitute (1) into (2),

$r^4 = 9$

$r^2 = 3$

$r = \sqrt{3}$ (> 1) or $-\sqrt{3}$ … (3)

$\therefore$ I is NOT true.

Substitute (3) into (1), we get

$a_1 = 7$

$\because$ r is irrational.

$\therefore$ Some of the terms of the sequence are irrational numbers. e.g. $a_2 = \pm 7\sqrt{3}$

$\therefore$ II is true.

For $r = \sqrt{3}$,

S(99)

$= \frac{7\left[(\sqrt{3})^{99} - 1\right]}{\sqrt{3} - 1}$

$\approx 3.963318044 \times 10^{24}$

$> 3 \times 10^{24}$

For $r = -\sqrt{3}$,

S(99)

$= \frac{7\left[(-\sqrt{3})^{99} - 1\right]}{-\sqrt{3} - 1}$

$\approx 1.061967869 \times 10^{24}$

$< 3 \times 10^{24}$

$\therefore$ III may NOT be true.

37. A

From the figure, when $x = 0$, $y = -2$.

$-2 = a \cos 2(0)^{\circ}$

$a = -2$

Substitute $(b, 2)$ into $y = -2 \cos 2x^{\circ}$,

$2 = -2 \cos 2b^{\circ}$

$b = 90$

38. B

$5\sin^2\theta + \sin\theta - 4 = 0$

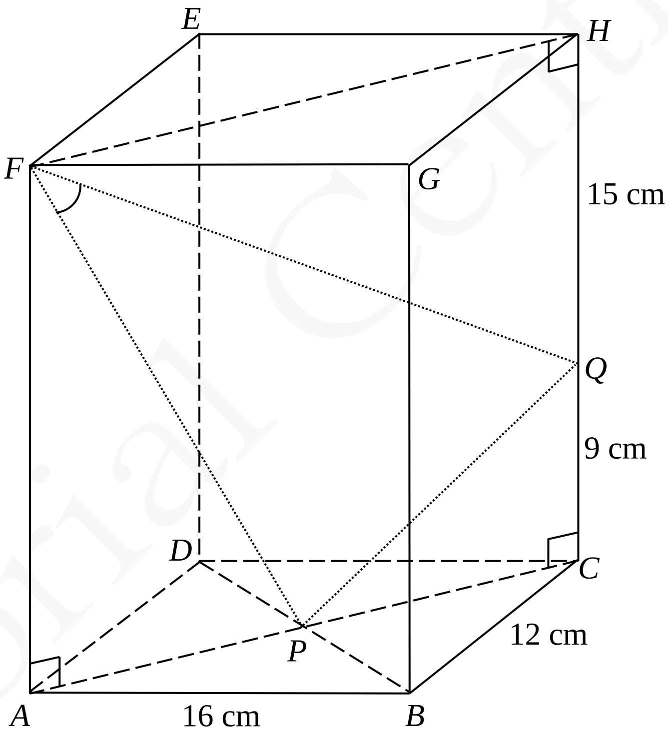
$(5\sin\theta - 4)(\sin\theta + 1) = 0$

$\sin\theta = \frac{4}{5}$ or -1

$\theta = 53.1^\circ, 127^\circ$ or 270°

$\therefore$ There are 3 roots.

39. A



By Pythagoras' theorem,

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{16^2 + 12^2}$

$= 20$ cm

$PC = AP = \frac{1}{2}AC = 10$ cm

$FH = AC = 20$ cm

By Pythagoras' theorem,

$FQ^2 = FH^2 + HQ^2$

$= 20^2 + 15^2$

$= 25^2$

By Pythagoras' theorem,

$PQ^2 = QC^2 + PC^2$

$= 9^2 + 10^2$

$= 181$

By Pythagoras' theorem,

$FP^2 = FA^2 + AP^2$

$= (15 + 9)^2 + 10^2$

$= 26^2$

By cosine formula,

$PQ^2 = FP^2 + FQ^2 - 2(FP)(FQ)\cos\angle PFQ$

$181 = 26^2 + 25^2 - 2(26)(25)\cos\angle PFQ$

$\cos\angle PFQ = \frac{56}{65}$

$\sin^2\angle PFQ + \cos^2\angle PFQ = 1$

$\sin^2\angle PFQ + \left(\frac{56}{65}\right)^2 = 1$

$\sin\angle PFQ = \frac{33}{65}$

40. D

Let G be the centre of the circle.

$\angle GBP = \angle GDP = 90^\circ$ (tangent $\perp$ radius)

$\angle GBP + \angle GDP + \angle BGD + \angle BPD = 360^\circ$ ($\angle$ sum of polygon)

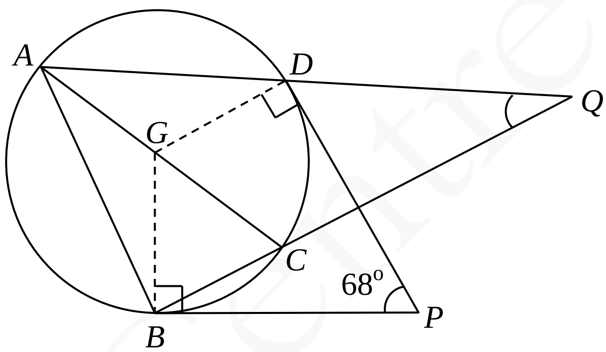
$90^\circ + 90^\circ + \angle BGD + 68^\circ = 360^\circ$

$\angle BGD = 112^\circ$

$\angle BAD = \frac{1}{2}\angle BGD$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \frac{1}{2}(112^\circ)$

$= 56^\circ$



$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)

$\angle AQB + \angle ABQ + \angle BAQ = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle AQB + 90^\circ + 56^\circ = 180^\circ$

$\angle AQB = 34^\circ$

41. D

$$\begin{cases} 2x - y - 6 = 0 \text{ i.e. } x = \frac{y+6}{2} \quad \ldots (1) \\ x^2 + y^2 - 8y - 14 = 0 \quad \ldots (2) \end{cases}$$

Substitute (1) into (2),

$\left(\frac{y+6}{2}\right)^2 + y^2 - 8y - 14 = 0$

$\frac{y^2+12y+36}{4} + y^2 - 8y - 14 = 0$

$y^2 - 4y - 4 = 0$

$\therefore$ The y-coordinate of the mid-point of PQ

$= -\frac{(-4)}{2}$ [i.e. $\frac{1}{2}$ × sum of roots]

$= 2$

42. A

The required probability

= P("2 cans of tea") + P("3 cans of tea")

$= \frac{C_2^3 C_2^9}{C_4^{9+3}} + \frac{C_3^3 C_1^9}{C_4^{9+3}}$

$= \frac{13}{55}$

<u>Alternatively</u>

The required probability

= 1 – [P("no tea") + P("1 can of tea")]

$= 1 - [\frac{C_4^9}{C_4^{9+3}} + \frac{C_1^3 C_3^9}{C_4^{9+3}}]$

$= \frac{13}{55}$

43. D

Number of committees that can be formed

$= C_6^{20} + C_5^{20}C_1^{15} + C_4^{20}C_2^{15}$

= 780 045

44. B

The upper quartile, $Q_3 = \frac{70+70}{2} = 70$

∴ I is NOT true.

The standard deviation of the distribution ≈ 11.57529697 < 12

∴ III is NOT true.

The mean of the distribution, $\mu = 63.25$

The standard score of Ada

$\approx \frac{85-63.25}{11.57529697}$

≈ 1.879001468 < 2

∴ II is true.

45. C

Note that adding 9 to each number of the set has no effect on the variance.

New variance

$= 4^2 \times 49$

= 784