Suggested Solution for 2018 HKDSE Mathematics(core) Multiple Choice Questions

1. B

$\frac{8^{2n+1}}{4^{3n+1}}$

$=\frac{(2^3)^{2n+1}}{(2^2)^{3n+1}}$

$=\frac{2^{6n+3}}{2^{6n+2}}$

$= 2^{(6n+3)-(6n+2)}$

$= 2$

2. D

$\frac{\alpha}{1-x} = \frac{\beta}{x}$

$\alpha x = \beta(1-x)$

$= \beta - \beta x$

$\alpha x + \beta x = \beta$

$(\alpha + \beta)x = \beta$

$x = \frac{\beta}{\alpha+\beta}$

3. C

$h^2 - 6h - 4k^2 - 12k$

$= h^2 - 4k^2 - 6h - 12k$

$= (h + 2k)(h - 2k) - 6(h + 2k)$

$= (h + 2k)(h - 2k - 6)$

4. A

$\frac{1}{3x+7} - \frac{1}{3x-7}$

$= \frac{3x-7-(3x+7)}{(3x+7)(3x-7)}$

$= \frac{-14}{9x^2-49}$

$= \frac{14}{49-9x^2}$

5. A

$y = 16 - (x - 6)^2$

$= 16 - (x^2 - 12x + 36)$

$= -x^2 + 12x - 20$

When $y = 0$, $-x^2 + 12x - 20 = 0$

$x^2 - 12x + 20 = 0$

$(x - 2)(x - 10) = 0$

$x = 2$ or 10

$\therefore$ A is true.

Note that

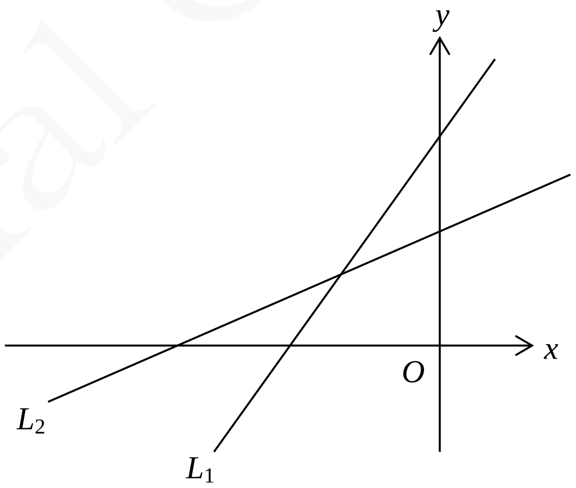
(i) $a = -1 < 0$ $\therefore$ The graph opens downwards. i.e. B is NOT true.

(ii) The y-intercept of the graph is $-20 \neq 16$. i.e. C is NOT true.

(iii) The graph does not pass through (0, 0). i.e. D is NOT true.

6. D

Rewrite $L_1 : y = -\frac{3}{a}x + \frac{b}{a}$ and $L_2 : y = -cx + d$.



From the figure, the slope of $L_1 = -\frac{3}{a} > 0$ ➔ $a < 0$

From the figure, the slope of $L_2 = -c > 0$ ➔ $c < 0$

Also, $-\frac{3}{a} > -c$

$\frac{3}{a} < c$

$3 > ac$ ($\because$ $a < 0$) i.e. $ac < 3$

$\therefore$ I is true.

From the figure, the y-intercept of L_1 > the y-intercept of L_2

$\frac{b}{a} > d$

$b < ad$ ($\because$ $a < 0$) i.e. $ad > b$

$\therefore$ II is NOT true.

Substitute $y = 0$ into the equations of L_1 and L_2, we get

the x-intercept of $L_1 = \frac{b}{3}$ and the x-intercept of $L_2 = \frac{d}{c}$

From the figure, the x-intercept of L_1 > the x-intercept of L_2

$\frac{b}{3} > \frac{d}{c}$

$bc < 3d$ ($\because$ $c < 0$)

$\therefore$ III is true.

7. D

$f(2m - 1)$
$= 3(2m - 1)^2 - 2(2m - 1) + 1$
$= 3(4m^2 - 4m + 1) - 4m + 2 + 1$
$= 12m^2 - 12m + 3 - 4m + 3$
$= 12m^2 - 16m + 6$

8. C

$\because$ g(x) is divisible by $x - 1$
$\therefore$ g(1) = 0
$(1)^8 + a(1)^7 + b = 0$
$b = -1 - a$

By the Remainder theorem, the required remainder = g(–1)
$= (-1)^8 + a(-1)^7 + b$
$= 1 - a + b$
$= 1 - a + (-1 - a)$
$= 1 - a - 1 - a$
$= -2a$

9. D

The required interest = $\$100\,000(1 + \frac{2\%}{12})^{3 \times 12} - \$100\,000$
$= \$6\,178$

10. B

$3a = 4b \Rightarrow \frac{a}{b} = \frac{4}{3}$ i.e. $a : b = 4 : 3$

$a : c = 2 : 5 \Rightarrow a : c = 4 : 10$

$\therefore$ $a : b : c = 4 : 3 : 10$

Let $a = 4k$, $b = 3k$ and $c = 10k$ where k is a constant.

$\frac{a+3b}{b+3c}$

$= \frac{4k+3(3k)}{3k+3(10k)}$

$= \frac{4k+9k}{3k+30k}$

$= \frac{13}{33}$

11. D

$w = \frac{k\sqrt{u}}{v^2}$ where k is a constant.

$k = \frac{wv^2}{\sqrt{u}}$

$k^2 = \frac{w^2v^4}{u}$ must be a constant.

12. A

$a_5 = a_3 + a_4 = 21 + a_4$

$a_6 = a_4 + a_5 = a_4 + (21 + a_4) = 89$

$2a_4 + 21 = 89$

$a_4 = 34$

$a_4 = a_2 + a_3 = 34$

i.e. $a_2 + 21 = 34$

$a_2 = 13$

$a_3 = a_1 + a_2 = 21$

i.e. $a_1 + 13 = 21$

$\therefore a_1 = 8$

13. C

$\frac{1-2x}{3} \geq x - 3$ or $4x + 9 < 1$

$1 - 2x \geq 3(x - 3)$ or $4x < -8$

$1 - 2x \geq 3x - 9$ or $x < -2$

$1 + 9 \geq 3x + 2x$ or $x < -2$

$5x \leq 10$ or $x < -2$

$x \leq 2$ or $x < -2$

$\therefore x \leq 2$

14. B

Absolute error of the measurement = 0.5 cm

The smallest possible area of the octagon

= The smallest possible area of rectangle *ABCD* – the largest possible area of rectangle *EFGH*

$= (6 - 0.5) \times (4 - 0.5) - (2 + 0.5) \times (2 + 0.5)$

$= 13 \text{ cm}^2$

The largest possible area of the octagon

= The largest possible area of rectangle *ABCD* – the smallest possible area of rectangle *EFGH*

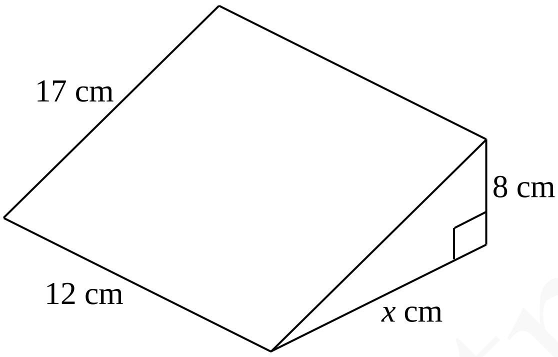
$= (6 + 0.5) \times (4 + 0.5) - (2 - 0.5) \times (2 - 0.5)$

$= 27 \text{ cm}^2$

$\therefore 13 < x < 27$

15. D

Consider the figure on the left.



By Pythagoras' Theorem,

$x^2 + 8^2 = 17^2$

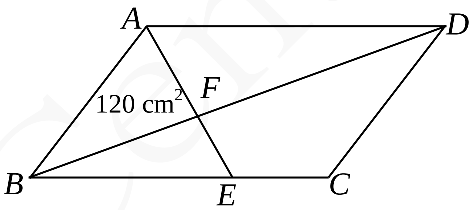
$x = 15$

Volume required

$= \frac{1}{2} \times 15 \times 8 \times 12$

$= 720 \text{ cm}^3$

16. A



∵ $BE : EC = 5 : 3$

∴ $EB : AD = 5 : 8$

Note that △*AFD* ~△*EFB*.

∴ $EF : AF = BF : DF = 5 : 8$ (corr. sides, ~△s)

∴ Area of △*BEF* : area of △*BAF* = *EF* : *AF* = 5 : 8 (∵ △*BEF* and △*BAF* have the same height.)

Area of △*BEF* : 120 = 5 : 8

Area of △*BEF* = 75 cm²

Area of △*ABE* = area of △*BEF* + area of △*BAF*

= 75 + 120

= 195 cm²

Area of △*DBC* : area of △*ABE* = *BC* : *BE* = 8 : 5 (∵ △*DBC* and △*ABE* have the same height.)

Area of △*DBC* : 195 = 8 : 5

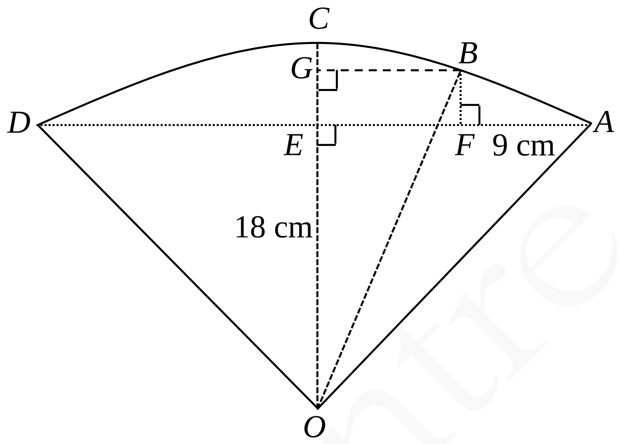
Area of △*DBC* = 312 cm²

∴ Area of *CDFE* = area of △*DBC* – area of △*BEF*

= 312 – 75

= 237 cm²

17. B



$\because$ $DE = AE$ (line from centre $\perp$ chord bisects chord)

$\therefore$ $DE = AE = \frac{AF+DF}{2}$

$= \frac{9+39}{2}$

$= 24$ cm

By Pythagoras' Theorem,

$OA^2 = AE^2 + OE^2$

$= 24^2 + 18^2$

$\therefore$ Radius, r $= OA = OB = 30$ cm

Let G be the foot of the perpendicular from B to OC.

$EF = AE - AF$

$= 24 - 9$

$= 15$ cm

$BG = EF = 15$ cm

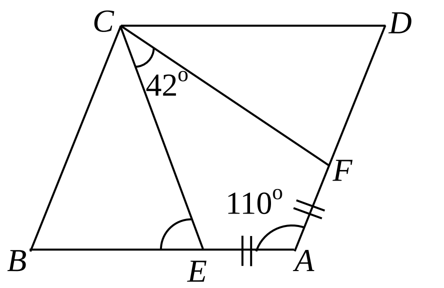
$\sin\angle BOG = \frac{BG}{OB} = \frac{15}{30} = \frac{1}{2}$

$\angle BOG = 30^\circ$

$\therefore$ Area of the sector $OBC = \pi \times 30^2 \times \frac{30^\circ}{360^\circ}$

$= 75\pi$ cm^2

18. B



In $\triangle BCE$ and $\triangle DCF$,

$BC = DC$ (property of rhombus)

$\angle CBE = \angle CDF$ (property of rhombus)

$\because$ $AB = AD$ (property of rhombus) and $AE = AF$ (given)

$\therefore$ $BE = AB - AE = AD - AF = DF$

$\therefore$ $\triangle BCE \cong \triangle DCF$ (SAS)

Join CA.

In $\triangle AEC$ and $\triangle AFC$,

$CE = CF$ (corr. sides, $\cong\triangle$s)

$CA = CA$ (common)

$AE = AF$ (given)

$\triangle AEC \cong \triangle AFC$ (SSS)

$\angle ECA = \angle FCA$ (corr. $\angle$s, $\cong\triangle$s)

$= 42^\circ \div 2 = 21^\circ$

Similarly, $\angle EAC = \angle FAC = 110^\circ \div 2 = 55^\circ$

$\therefore$ $\angle BEC = \angle ECA + \angle EAC$ (ext. $\angle$ of $\triangle$)

$= 21^\circ + 55^\circ = 76^\circ$

Alternatively

$\angle BCE = \angle DCF$

$= (110^\circ - 42^\circ) \div 2$

$= 34^\circ$

$\angle CBE + \angle BAD = 180^\circ$ (int. $\angle$s, BC // AD)

$\angle CBE + 110^\circ = 180^\circ$

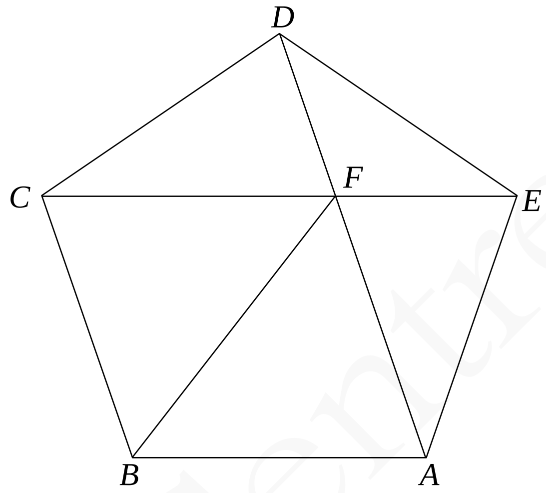
$\angle CBE = 70^\circ$

$\angle BEC + \angle CBE + \angle BCE = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle BEC + 70^\circ + 34^\circ = 180^\circ$

$\angle BEC = 76^\circ$

19. D



$\because$ $DC = DE$

$\therefore$ $\angle DCE = \angle DEC$ (base $\angle$s, isos. $\triangle$)

Note $\angle CDE = 108^o$.

$\angle DCE + \angle DEC + \angle CDE = 180^o$ ($\angle$ sum of $\triangle$)

$2\angle DCE + 108^o = 180^o$

$\angle DCE = 36^o$

Similarly, $\angle EDA = \angle EAD = 36^o$.

Then, $\angle CDF = 108^o - 36^o = 72^o$

$\angle CFD = \angle FDE + \angle DEF$ (ext. $\angle$ of $\triangle$)

$\angle CFD = 36^o + 36^o = 72^o$

$\because$ $\angle CDF = \angle CFD = 72^o$

$\therefore$ $CD = CF$ (sides opp. equal $\angle$s)

$\therefore$ I is true.

Similarly, $AF = AE$.

Then, $AF = AE = CD = CF$.

In $\triangle ABF$ and $\triangle CBF$,

$AF = CF$ (proved)

$BF = BF$ (common)

$AB = CB$ (given)

$\therefore$ $\triangle ABF \cong \triangle CBF$ (SSS)

$\therefore$ II is true.

$\angle EAF = 36^o$

$\angle BAF = 108^o - 36^o = 72^o$

$\because$ $AF = AB$

$\therefore$ $\angle ABF = \angle AFB$ (base $\angle$s, isos. $\triangle$)

$\angle ABF + \angle AFB + \angle BAF = 180^o$ ($\angle$ sum of $\triangle$)

$2\angle AFB + 72^o = 180^o$

$\angle AFB = 54^o$

$\therefore$ $\angle AFB + \angle EAF = 54^o + 36^o = 90^o$

$\therefore$ III is true.

20. B

By Pythagoras' Theorem,

$BF^2 + BE^2 = EF^2$

$BF^2 + 4^2 = 5^2$

$BF = 3$ cm

Note that $\triangle EBF \sim \triangle EAD$.

Let $AB = AD = x$ cm.

$\frac{BF}{AD} = \frac{EB}{EA}$ (corr. sides, ~△s)

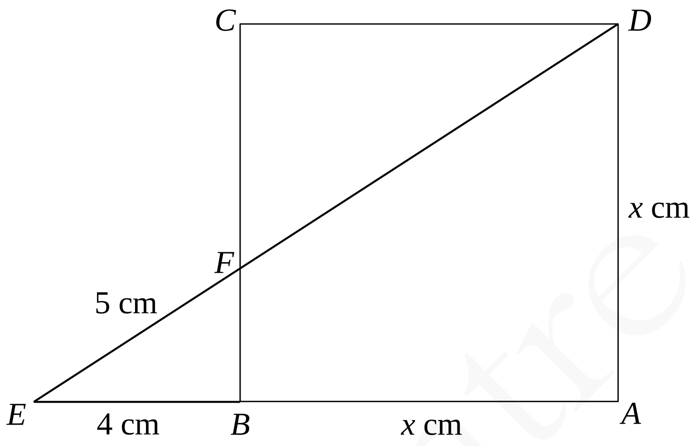
$\frac{3}{x} = \frac{4}{4+x}$

$x = 12$

$\frac{EF}{ED} = \frac{BF}{AD}$ (corr. sides, ~△s)

$\frac{5}{5+DF} = \frac{3}{12}$

$DF = 15$ cm



Alternatively

$CF = BC - BF = 12 - 3 = 9$ cm

By Pythagoras' Theorem,

$DF^2 = CF^2 + CD^2$

$= 9^2 + 12^2$

$DF = 15$ cm

21. C

Let $AE = EF = FB = x$.

$BE = CE \cos\alpha = 2x$

$AF = DF \cos\beta = 2x$

$\therefore$ $CE \cos\alpha = DF \cos\beta$

$\therefore$ II is true.

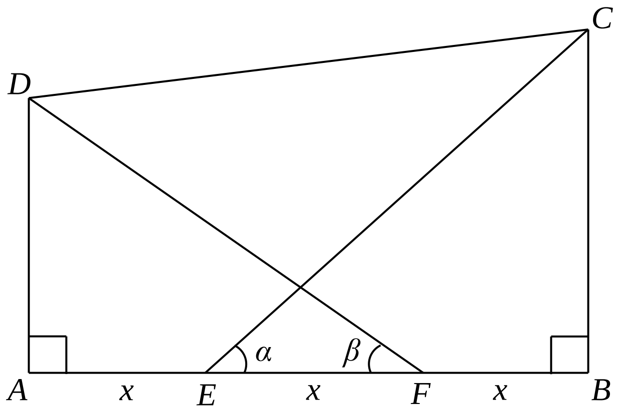
$\frac{AD}{AF} = \tan\beta$ i.e. $AF = \frac{AD}{\tan\beta} = 2x$

$\frac{BC}{BE} = \tan\alpha$ i.e. $BE = \frac{BC}{\tan\alpha} = 2x$

$\therefore$ $\frac{AD}{\tan\beta} = \frac{BC}{\tan\alpha}$

i.e. $AD \tan\alpha = BC \tan\beta$

$\therefore$ III is true.



22. B

Let $\angle CED = x$.

Then, $\angle DBE = \angle CED = x$ (base $\angle$s, isos. $\triangle$)

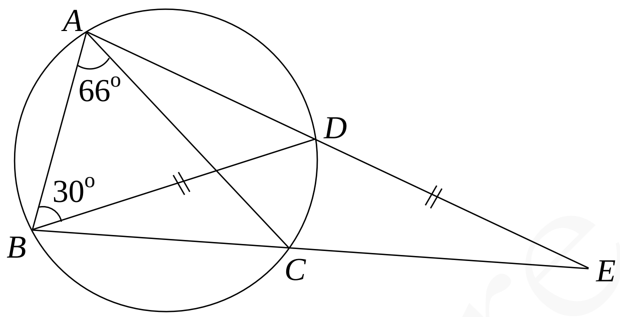
$\angle DAC = \angle DBC = x$ ($\angle$s in the same segment)

Now, consider $\triangle ABE$.

$\angle ABE + \angle BAE + \angle AEB = 180^\circ$ ($\angle$ sum of $\triangle$)

$(30^\circ + x) + (66^\circ + x) + x = 180^\circ$

$x = 28^\circ$



23. B

The figure repeats itself 4 times when rotated about an axis at the centre of the figure in one revolution.

24. A

By Pythagoras' Theorem,

$CD^2 = 12^2 + 16^2$

$CD = 20$

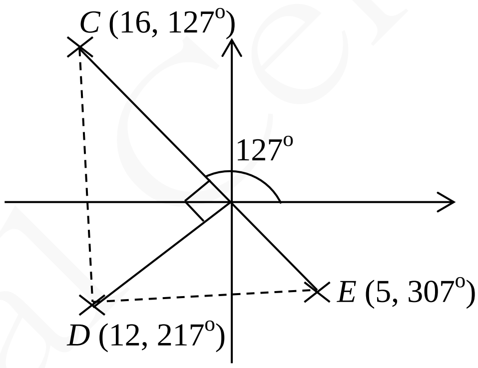
$DE^2 = 12^2 + 5^2$

$DE = 13$

Perimeter of $\triangle CDE$

$= 5 + 16 + 20 + 13$

$= 54$



25. D

Rewrite $L_1 : y = 3x + 7$ and $L_2 : y = 3x - \frac{11}{4}$

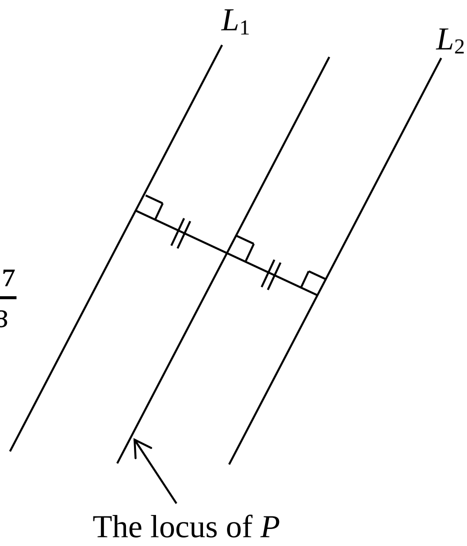
Slope of L_1 = slope of $L_2 = 3$ i.e. $L_1 // L_2$

The locus of P is a straight line parallel to both L_1 and L_2 and sits in the middle of L_1 and L_2.

The y-intercept of the equation of the locus of $P = (-\frac{11}{4} + 7) \div 2 = \frac{17}{8}$

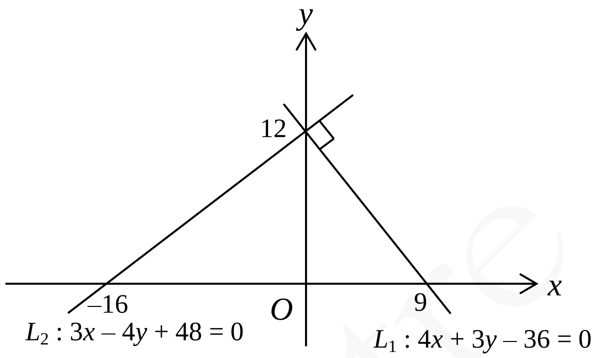
The required equation is $y = 3x + \frac{17}{8}$

i.e. $24x - 8y + 17 = 0$



26. C

Rewrite $L_1 : y = -\frac{4}{3}x + 12$



$\therefore$ The slope of $L_1 = -\frac{4}{3}$ and the y-intercept of $L_1 = 12$

Substitute $y = 0$ into the equation of L_1, we get $x = 9$.

$\because$ $L_2 \perp L_1$

$\therefore$ The slope of $L_2 = \frac{3}{4}$ and the y-intercept of $L_2 = 12$

The equation of L_2 is $y = \frac{3}{4}x + 12$. i.e. $3x - 4y + 48 = 0$

Substitute $y = 0$ into the equation of L_2, we get $x = -16$.

The required area $= \frac{1}{2} \times [9 - (-16)] \times 12$

$= 150$

27. C

Rewrite the equation of the circle C as $x^2 + y^2 - 6x + 2y + \frac{6}{5} = 0$.

Radius of C, r $= \sqrt{\left(\frac{-6}{2}\right)^2 + \left(\frac{2}{2}\right)^2 - \frac{6}{5}}$

$= \sqrt{\frac{44}{5}}$

Circumference of $C = 2\pi r$

$= 2\pi\sqrt{\frac{44}{5}}$

≈ 18.63893975

< 20

Note that :

The coordinates of the centre of C are $(-\frac{-6}{2}, -\frac{2}{2})$ i.e. $(3, -1)$ ➔ D is not true.

$\because$ The centre lies in the third quadrant.

$\therefore$ C cannot only lie in the second quadrant. ➔ B is not true.

The distance between the origin and the centre of $C = \sqrt{3^2 + (-1)^2} = \sqrt{10} > r$

$\therefore$ The origin lies outside C. ➔ A is not true.

28. A

The favourable outcomes are (1, 4), (1, 4), (1, 4), (2, 3) and (2, 3).

Number of possible outcomes = C_2^7 = 21

∴ The required probability = $\frac{5}{21}$

<u>Alternatively</u>

Refer to the table below. 1st card

2nd card	1	1	1	2	2	3
4	⑤	⑤	⑤	6	6	7
3	4	4	4	⑤	⑤	
2	3	3	3	4		
2	3	3	3			
1	2	2				
1	2					

Number of favourable outcomes = 5

Number of possible outcomes = 21

∴ The required probability = $\frac{5}{21}$

29. C

Let x be the required mean. Then,

$4x + 6 \times 108 = 10 \times 132$

$x = 168$

30. A

The first quartile, $Q_1 = 30 + a$

The third quartile, $Q_3 = 60 + b$

The inter-quartile range = $Q_3 - Q_1$

$= (60 + b) - (30 + a)$

$= 30 + b - a \leq 25$ i.e. $a - b \geq 5$

∵ $b \geq 0$ and $a \leq 9$

∴ $5 \leq a \leq 9$ and $0 \leq b \leq 4$

31. C

Observing that the graph on the right [which is f(x)] is reflected about the x-axis and translated 4 units to the left to give the graph on the left, the answer is C.

Note that

A represents a graph due to reflection about the x-axis and enlargement along y-direction.

B represents a graph due to reflection about the y-axis and contraction along x-direction.

D represents a graph due to reflection about the y-axis and translation along x-direction.

32. C

∵ y increases as x increases in both graphs.

∴ $a > 1$ and $b > 1$

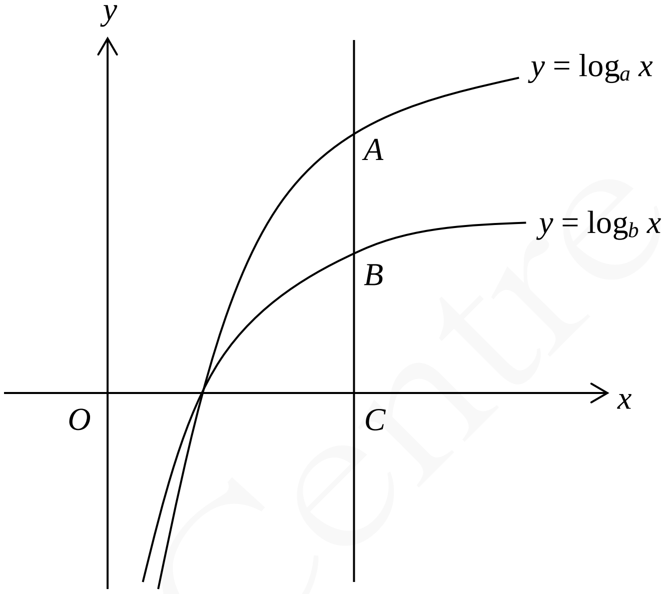
∴ I is true.

For $x > 1$, $\log_a x > \log_b x$

$\frac{\log x}{\log a} > \frac{\log x}{\log b}$ [By change of base]

$\frac{\log b}{\log a} > 1 \Rightarrow b > a$

∴ II is NOT true.



From the graphs,

$$\frac{AB}{BC} = \frac{AC - BC}{BC}$$

$$= \frac{AC}{BC} - 1$$

$$= \frac{\log_a OC}{\log_b OC} - 1$$

$$= \frac{\log OC / \log a}{\log OC / \log b} - 1$$

$$= \frac{\log b - \log a}{\log a}$$

$$= \log_a \frac{b}{a}$$

∴ III is true.

33. D

$y = kx^a$

$\log_4 y = \log_4(kx^a)$

$= \log_4 x^a + \log_4 k$

$= a\log_4 x + \log_4 k$

Substitute (1, 2) into the equation,

$2 = a(1) + \log_4 k$ i.e. $a + \log_4 k = 2$ … (1)

Substitute (9, 6) into the equation,

$6 = a(9) + \log_4 k$ i.e. $9a + \log_4 k = 6$ … (2)

(1) × 9 – (2),

$8\log_4 k = 12$

$k = 4^{\frac{3}{2}}$

$= 8$

Alternatively

$\begin{cases} \log_4 x = 1 \Rightarrow x = 4 \\ \log_4 y = 2 \Rightarrow y = 4^2 \end{cases}$

∴ $4^2 = k(4)^a$ i.e. $k(4^a) = 4^2$ … (1)

$\begin{cases} \log_4 x = 9 \Rightarrow x = 4^9 \\ \log_4 y = 6 \Rightarrow y = 4^6 \end{cases}$

∴ $4^6 = k(4^9)^a$ i.e. $k(4^{9a}) = 4^6$ … (2)

From (1),

$[k(4^a)]^9 = (4^2)^9$

$k^9(4^{9a}) = 4^{18}$ … (3)

(3) ÷ (2),

$k^8 = 4^{12}$

$k = 4^{\frac{3}{2}} = 8$

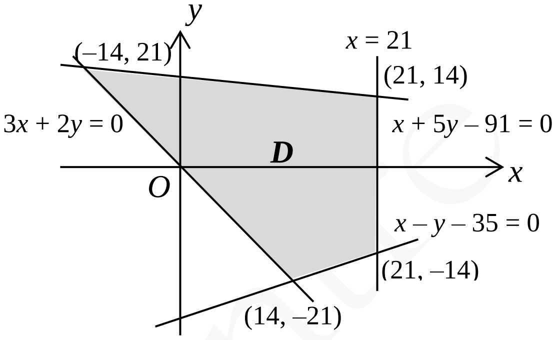
34. C

Draw the straight lines of $x = 21$, $x - y - 35 = 0$, $x + 5y - 91 = 0$ and $3x + 2y = 0$ respectively.

Shade the region D.

The points of intersections are (–14, 21), (14, –21), (21, –14), (21, 14).

Let $P = 5x + 6y + 234$.



$P(-14, 21) = 5(-14) + 6(21) + 234 = 290$

$P(14, -21) = 5(14) + 6(-21) + 234 = 178$

$P(21, -14) = 5(21) + 6(-14) + 234 = 255$

$P(21, 14) = 5(21) + 6(14) + 234 = 423$

$\therefore$ The least value of $5x + 6y + 234$ is 178.

35. B

Let $S(n) = 6n^2 - n$ and $T(n)$ be the nth term of the sequence. Then,

$T(n) = S(n) - S(n - 1)$

$= 6n^2 - n - [6(n - 1)^2 - (n - 1)]$

$= 12n - 7$

When $T(n) = 22$ i.e. $12n - 7 = 22$ ➔ $n = \frac{29}{12}$ which is not an integer.

$\therefore$ I is NOT true.

$T(1) = 12(1) - 7 = 5$

$\therefore$ II is true.

<u>Alternatively</u>

$S(1) = 6(1)^2 - 1 = 5$

$\therefore$ II is true.

$\frac{T(2)}{T(1)} = \frac{12(2)-7}{12(1)-7} = \frac{17}{5}$

$\frac{T(3)}{T(2)} = \frac{12(3)-7}{12(2)-7} = \frac{29}{17} \neq \frac{T(2)}{T(1)}$

$\therefore$ III is NOT true.

36. A

Note that m and n are the roots of the quadratic equation $2x^2 + 5x - 14 = 0$.

$m + n = -\frac{5}{2}$ and $mn = -7$

$(m + 2)(n + 2)$

$= mn + 2(m + n) + 4$

$= -7 + 2(-\frac{5}{2}) + 4$

$= -8$

37. D

$\frac{2i^{12}+3i^{13}+4i^{14}+5i^{15}+6i^{16}}{1-i}$

$=\frac{2+3i+4(-1)+5(-i)+6}{1-i}$ [Note : $i^{4n}=1$, $i^{4n+1}=i$, $i^{4n+2}=-1$, $i^{4n+3}=-i$]

$=\frac{4-2i}{1-i}$

$=\frac{4-2i}{1-i}\times\frac{1+i}{1+i}$

$=\frac{4-2i+4i-2i^2}{1+1}$

$=\frac{6+2i}{2}$

$= 3 + i$

∴ The real part of the complex number is 3.

38. B

$6\cos^2 x = \cos x + 5$

$6\cos^2 x - \cos x - 5 = 0$

$(6\cos x + 5)(\cos x - 1) = 0$

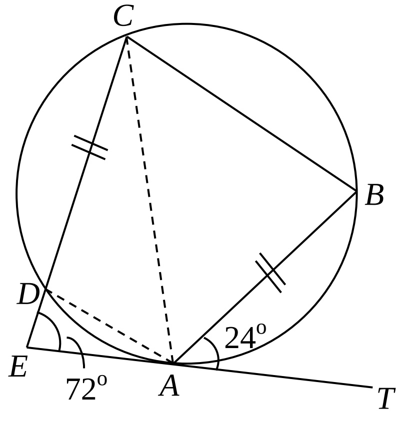
$\cos x = 1$ or $-\frac{5}{6}$

$x = 0^\circ, 146^\circ$ or 214°

∴ The equation has 3 roots.

39. B

Join AC and AD.



$\angle ACB = \angle BAT$ ($\angle$ in alt. segment)

$= 24^\circ$

$\overset{\frown}{CD} = \overset{\frown}{AB}$ (equal chords, equal arcs)

$\angle DAC = \angle ACB$ (arcs prop. to $\angle$s at ⊙ce)

$= 24^\circ$

Let $\angle DAE = x$. Then,

$\angle DCA = \angle DAE = x$ ($\angle$ in alt. segment)

$\angle AED + \angle DCA + \angle DAC + \angle DAE = 180^\circ$ ($\angle$ sum of △)

$72^\circ + x + 24^\circ + x = 180^\circ$

$x = 42^\circ$

$\angle ABC = \angle EAC$ ($\angle$ in alt. segment)

$= \angle DAC + \angle DAE$

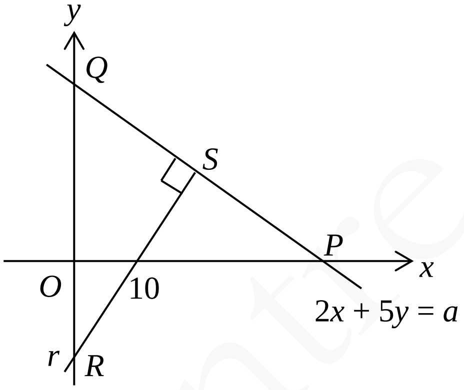
$= 24^\circ + 42^\circ$

$= 66^\circ$

40. A

Let S be the foot of the perpendicular from R to PQ.

Note that the intersection of OP and RS is the orthocenter of $\triangle PQR$.



The slope of $PQ = -\frac{2}{5}$.

Let the y-coordinate of R be r.

The slope of $RS = -\frac{r}{10}$.

$\because \quad PQ \perp RS$

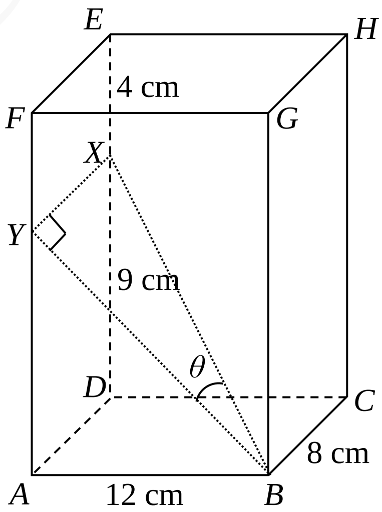
$\therefore \quad \left(-\frac{2}{5}\right)\left(-\frac{r}{10}\right) = -1$

$r = -25$

41. D

Let Y be the projection of X on the plane $ABGF$. Then, $XY \perp YB$ and $\theta = \angle YBX$.

Note that $XY = BC = 8$ cm and $YA = XD = 9$ cm.



$YB^2 = YA^2 + AB^2 = 9^2 + 12^2$

$YB = 15$ cm

$XB^2 = YB^2 + XY^2$

$\quad = 15^2 + 8^2$

$XB = 17$ cm

$\therefore \quad \cos\theta = \frac{YB}{XB} = \frac{15}{17}$

42. A

Number of teams formed

$= C_3^{14} + C_3^{15}$

$= 819$

43. C

John gets a number '6' in the following situations.

He gets '6' in the 1st throw. Probability = $\frac{1}{6}$ or

He does not get '1' or '6' in the 1st throw and Mary does not get '1' or '6' in the 2nd throw and then he gets '6' in the 3rd throw. Probability = $\frac{4}{6}\times\frac{4}{6}\times\frac{1}{6}$ or

He gets '6' in the 5th throw. Probability = $\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}\times\frac{1}{6}$ etc.

The required probability

$= \frac{1}{6} + \frac{4}{6}\times\frac{4}{6}\times\frac{1}{6} + \frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}\times\frac{1}{6} + \ldots$ (sum to infinity)

$= \dfrac{\frac{1}{6}}{1-\left(\frac{4}{6}\right)^2}$

$= \frac{3}{10}$

44. B

Let σ be the standard deviation of the test scores. Then,

$\frac{46-68}{\sigma} = -2.2$

$\sigma = 10$

Susan's standard score

$= \frac{52-68}{10}$

$= -1.6$

45. A

∵ All the terms are in an arithmetic sequence

∴ Any consecutive 7 terms must have equal dispersion.

∴ The required variance is also 9.