

Suggested Solution for 2019 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$$\begin{aligned}
 & (a-b)(a^2+ab-b^2) \\
 &= a(a^2+ab-b^2) - b(a^2+ab-b^2) \\
 &= a^3 + a^2b - ab^2 - a^2b - ab^2 + b^3 \\
 &= a^3 - 2ab^2 + b^3
 \end{aligned}$$

2. D

$$\begin{aligned}
 & \frac{(6x^7)^2}{4x^5} \\
 &= \frac{6^2 x^{7 \times 2}}{4x^5} \\
 &= \frac{36x^{14}}{4x^5} \\
 &= 9x^9
 \end{aligned}$$

3. B

$$\begin{cases} 6x - 7y = 40 \dots (1) \\ 2x + 11y = 40 \dots (2) \end{cases}$$

$$(2) \times 3 - (1),$$

$$40y = 80$$

$$y = 2$$

4. C

$$(x-8)(x+\alpha) - 6 \equiv (x-9)^2 + \beta$$

Substitute $x = 8$ into both sides,

$$(8-8)(8+\alpha) - 6 = (8-9)^2 + \beta$$

$$-6 = 1 + \beta$$

$$\beta = -7$$

Alternatively

$$x^2 + (\alpha-8)x - 8\alpha - 6 \equiv x^2 - 18x + 81 + \beta$$

By comparing the coefficients of x ,

$$\alpha - 8 = -18$$

$$\alpha = -10$$

By comparing the constant terms,

$$81 + \beta = -8\alpha - 6 = -8(-10) - 6$$

$$\beta = -7$$

5. A

$$h = 3 - \frac{5}{k+4}$$

$$h(k+4) = 3(k+4) - 5$$

$$hk + 4h = 3k + 12 - 5$$

$$4h - 7 = -3k - hk$$

$$k(3-h) = 4h - 7$$

$$k = \frac{4h-7}{3-h}$$

6. D

$$x = 0.07(\text{correct to 2 decimal places})$$

$$x = 0.066(\text{correct to 2 significant figures})$$

$$x = 0.066(\text{correct to 3 decimal places})$$

$$x = 0.0656(\text{correct to 3 significant figures})$$

7. B

$$-2(x - 5) + 5 < 21 \text{ or } \frac{3x-5}{7} > 1$$

$$-2x + 10 + 5 < 21 \text{ or } 3x - 5 > 7$$

$$-2x < 21 - 15 \text{ or } 3x > 7 + 5$$

$$-2x < 6 \text{ or } 3x > 12$$

$$x > -3 \text{ or } x > 4$$

$$\therefore x > -3$$

\therefore The least integer is -2 .

8. C

$$f(c) + f(-c)$$

$$= (c)^3 + c(c)^2 + c + (-c)^3 + c(-c)^2 + c$$

$$= c^3 + c^3 + c - c^3 + c^3 + c$$

$$= 2c^3 + 2c$$

9. D

By Factor theorem,

$$2\left(-\frac{k}{2}\right)^4 + k\left(-\frac{k}{2}\right)^3 - 4\left(-\frac{k}{2}\right) - 16 = 0$$

$$\frac{k^4}{8} - \frac{k^4}{8} + 2k - 16 = 0$$

$$k = 8$$

10. A

$$y = (3 - x)(x + 2) + 6$$

$$= -x^2 + x + 6 + 6$$

$$= -x^2 + x + 12$$

$$\therefore a = -1 < 0$$

\therefore The graph opens downwards.

\therefore I is true.

Put $x = 1$,

$$-(1)^2 + (1) + 12 = 12 \neq 10$$

\therefore II is NOT true.

When $y = 0$,

$$-x^2 + x + 12 = 0$$

$$x^2 - x - 12 = 0$$

$$(x + 3)(x - 4) = 0$$

$$x = -3 \text{ or } 4$$

i.e. The x -intercepts of the graph are -3 and 4 .

\therefore III is NOT true.

11. C

$$\begin{aligned}\text{The required amount} &= \$65\,000\left(1 + \frac{7\%}{4}\right)^{8 \times 4} \\ &= \$113\,244(\text{correct to the nearest dollar})\end{aligned}$$

12. B

$$\frac{140x+315y}{x+y} = 210$$

$$140x + 315y = 210(x + y)$$

$$140x + 315y = 210x + 210y$$

$$210x - 140x = 315y - 210y$$

$$70x = 105y$$

$$\frac{x}{y} = \frac{105}{70} = \frac{3}{2}$$

$$\text{i.e. } x : y = 3 : 2$$

13. A

$$z = \frac{kx^2}{\sqrt{y}} \text{ where } k \text{ is a constant.}$$

$$\text{Change in } z, z' = \frac{k[(1-40\%)x]^2}{\sqrt{(1+44\%)y}}$$

$$= \frac{0.3kx^2}{\sqrt{y}}$$

$$= 0.3z$$

Percentage change in z

$$= \frac{0.3z - z}{z} \times 100\%$$

$$= -70\%$$

14. C

$$T(1) = 6$$

$$T(2) = 10$$

$$T(3) = 14$$

:

:

$$T(n) = 4n + 2$$

$$\therefore T(9) = 4(9) + 2$$

$$= 38$$

15. D

Let h cm be the height of the lateral surface.

By Pythagoras' theorem,

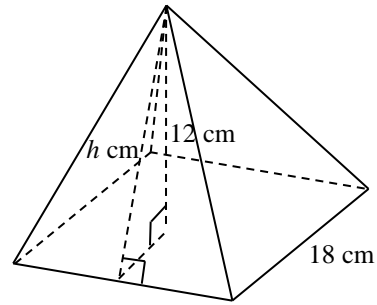
$$h = \sqrt{12^2 + 9^2}$$

$$= 15 \text{ cm}$$

The total surface area of the pyramid

$$= 18 \times 18 + \frac{1}{2} \times 18 \times 15 \times 4$$

$$= 864 \text{ cm}^2$$



16. D

Note that $\triangle BEX \sim \triangle BFY \sim \triangle DAX$.

$$\therefore EX : FY = BE : BF = 2 : 9 \text{ and}$$

$$EX : AX = BE : DA = 2 : 12 = 1 : 6$$

$\therefore \triangle BEX$ and $\triangle ABX$ have the same height.

$$\therefore \text{Area of } \triangle BEX : \text{area of } \triangle ABX = EX : AX$$

$$\text{Area of } \triangle BEX : 24 = 1 : 6$$

$$\text{Area of } \triangle BEX = 4 \text{ cm}^2$$

$$\therefore \triangle BEX \sim \triangle BFY$$

$$\therefore \frac{\text{Area of } \triangle BEX}{\text{Area of } \triangle BFY} = \left(\frac{EX}{FY}\right)^2$$

$$\text{i.e. } \frac{4}{\text{Area of } \triangle BFY} = \left(\frac{2}{9}\right)^2$$

$$\therefore \text{Area of } \triangle BFY = 81 \text{ cm}^2$$

$$\therefore \triangle BEX \sim \triangle DAX$$

$$\therefore \frac{\text{Area of } \triangle DAX}{\text{Area of } \triangle BEX} = \left(\frac{DA}{BE}\right)^2$$

$$\text{i.e. } \frac{\text{Area of } \triangle DAX}{4} = \left(\frac{6}{1}\right)^2$$

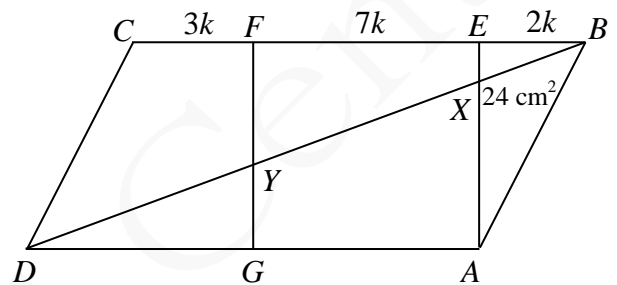
$$\text{Area of } \triangle DAX = 144 \text{ cm}^2$$

$$\text{Area of } CDYF = \text{Area of } \triangle BDC - \text{Area of } \triangle BFY$$

$$= \text{Area of } \triangle DAX + \text{Area of } \triangle ABX - \text{Area of } \triangle BFY$$

$$= 144 + 24 - 81$$

$$= 87 \text{ cm}^2$$



17. A

$$\angle BAD = \angle BDA \text{ (base } \angle \text{s, isos. } \triangle)$$

$$\text{Let } \angle BAD = \angle BDA = x.$$

$$\angle CBD = \angle BAD + \angle BDA \text{ (ext. } \angle \text{ of } \triangle)$$

$$= x + x$$

$$= 2x$$

$$\angle CDB = \angle CBD \text{ (base } \angle \text{s, isos. } \triangle)$$

$$= 2x$$

$$\angle BDA + \angle CDB + \angle CDE = 180^\circ \text{ (adj. } \angle \text{s on st. line)}$$

$$x + 2x + 66^\circ = 180^\circ$$

$$x = 38^\circ$$

$$\angle BAD + \angle ACD = \angle CDE \text{ (ext. } \angle \text{ of } \triangle)$$

$$38^\circ + \angle ACD = 66^\circ$$

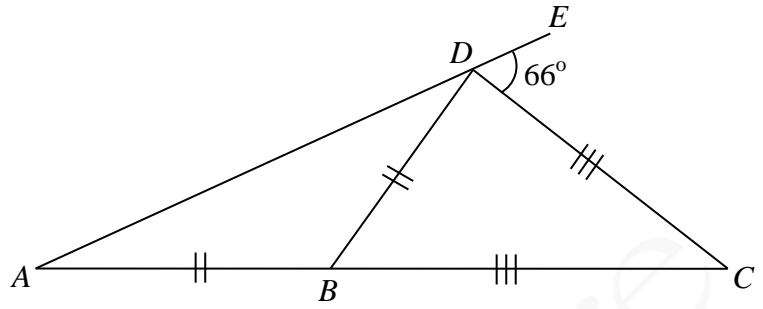
$$\angle ACD = 28^\circ$$

Alternatively

$$\angle CDB + \angle CBD + \angle ACD = 180^\circ \text{ (} \angle \text{ sum of } \triangle)$$

$$2(38^\circ) + 2(38^\circ) + \angle ACD = 180^\circ$$

$$\angle ACD = 28^\circ$$



18. D

Let $EB = x$ cm. Then, $AD = DE = 2x$ cm.

$$AC = AB = 5x \text{ cm}$$

$$\angle AEC = \angle ADF \text{ (corr. } \angle \text{s, } DF \parallel EC)$$

$$= 90^\circ$$

By Pythagoras' theorem,

$$AE^2 + EC^2 = AC^2$$

$$(4x)^2 + 60^2 = (5x)^2$$

$$x = 20$$

By mid-point theorem,

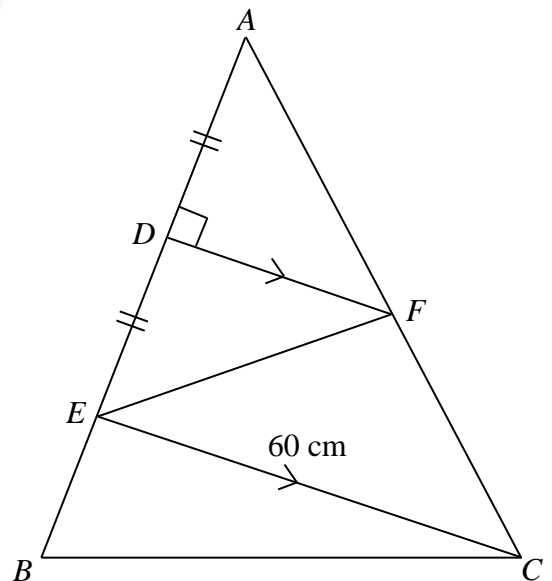
$$FD = \frac{1}{2}CE = 30 \text{ cm}$$

By Pythagoras' theorem,

$$EF^2 = FD^2 + DE^2$$

$$= 30^2 + 40^2$$

$$EF = 50 \text{ cm}$$



19. A

By Pythagoras' theorem,

$$AB^2 + BD^2 = AD^2$$

$$18^2 + BD^2 = 30^2$$

$$BD = 24 \text{ cm}$$

$$\angle CDB = \angle ABD = 90^\circ \text{ (alt. } \angle \text{s, } AB \parallel DC \text{)}$$

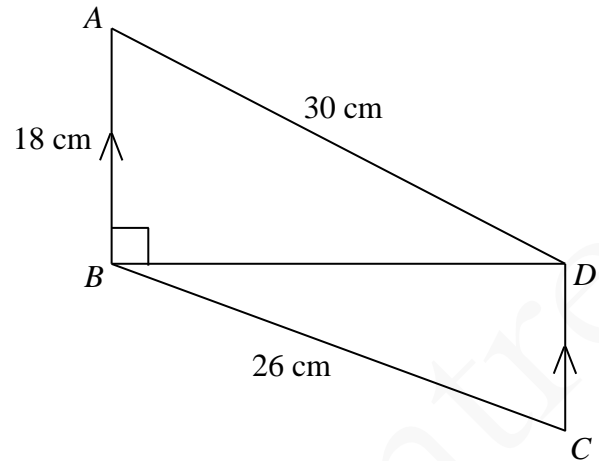
By Pythagoras' theorem,

$$DC^2 + BD^2 = BC^2$$

$$DC^2 + 24^2 = 26^2$$

$$DC = 10 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } ABCD &= \frac{1}{2} \times 18 \times 24 + \frac{1}{2} \times 10 \times 24 / \frac{1}{2} \times (18 + 10) \times 24 \\ &= 336 \text{ cm}^2 \end{aligned}$$



20. C

$$\angle EBF = \angle EFB \text{ (base } \angle \text{s, isos. } \triangle \text{)}$$

$$\angle EBF + \angle EFB + \angle BEF = 180^\circ$$

$$2\angle EBF + 56^\circ = 180^\circ$$

$$\angle EBF = 62^\circ$$

$$\angle DBC = \angle ABD \text{ (properties of rhombus)}$$

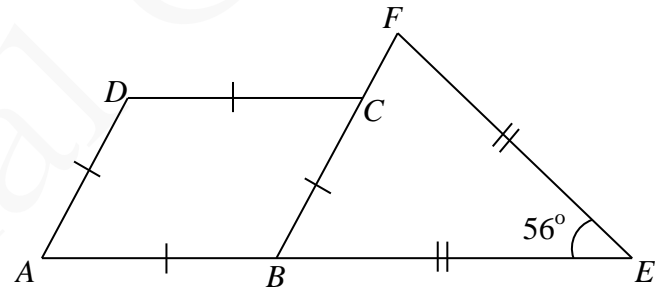
$$\angle DBC + \angle ABD + \angle EBF = 180^\circ \text{ (adj. } \angle \text{s on st. line)}$$

$$2\angle ABD + 62^\circ = 180^\circ$$

$$\angle ABD = 59^\circ$$

$$\angle BDC = \angle ABD \text{ (alt. } \angle \text{s, } AB \parallel DC \text{)}$$

$$= 59^\circ$$



21. B

$$\angle AOC = \angle BOD \text{ (equal chords, equal } \angle \text{s)}$$

$$\angle AOC = \angle AOD + \angle COD$$

$$= \angle AOD + 48^\circ$$

Similarly,

$$\angle BOD = \angle BOC + \angle COD$$

$$= \angle BOC + 48^\circ$$

$$\therefore \angle AOD + 48^\circ = \angle BOC + 48^\circ \text{ i.e. } \angle AOD = \angle BOC$$

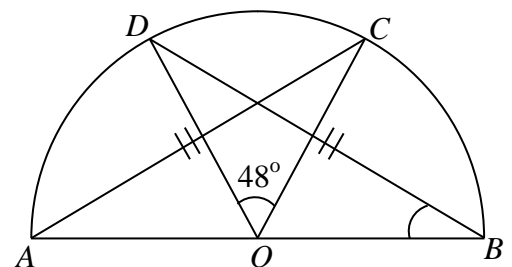
$$\angle AOD + \angle COD + \angle BOC = 180^\circ \text{ (adj. } \angle \text{s on st. line)}$$

$$2\angle AOD + 48^\circ = 180^\circ$$

$$\angle AOD = 66^\circ$$

$$\angle ABD = \frac{1}{2} \angle AOD \text{ (} \angle \text{ at centre twice } \angle \text{ at } \odot^{\text{ce}} \text{)}$$

$$= 33^\circ$$



22. B

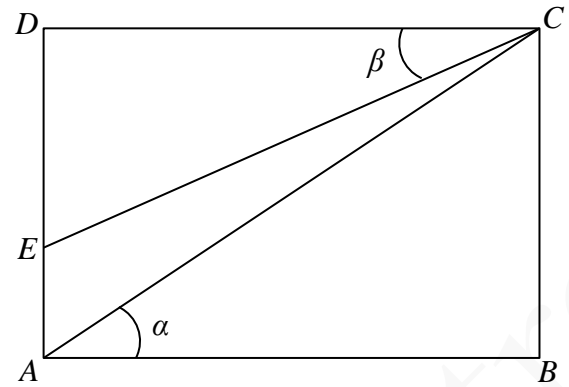
$$\frac{AB}{AC} = \cos \alpha \quad \text{i.e.} \quad AB = AC \cos \alpha$$

$$\frac{CD}{CE} = \cos \beta \quad \text{i.e.} \quad CD = CE \cos \beta$$

$$\therefore AB = CD$$

$$\therefore AC \cos \alpha = CE \cos \beta$$

$$\text{i.e.} \quad \frac{CE}{AC} = \frac{\cos \alpha}{\cos \beta}$$



23. A

Put $y = 0$,

$$x\text{-intercept} = -\frac{15}{a}$$

From the graph, $-\frac{15}{a} > 5$

$$\therefore -3 < a < 0$$

 \therefore II is true.Put $x = 0$,

$$y\text{-intercept} = -\frac{15}{b}$$

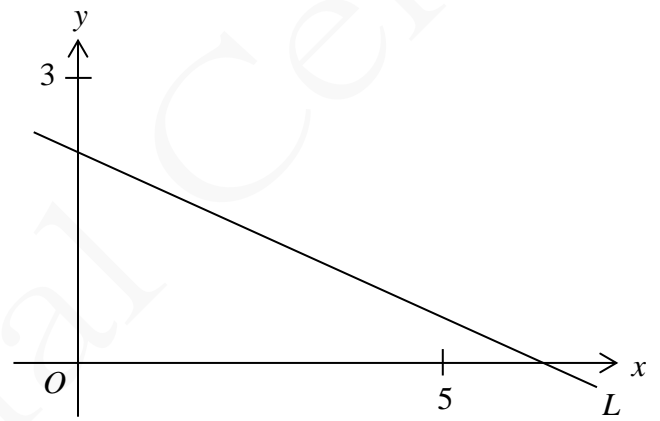
From the graph, $0 < -\frac{15}{b} < 3$

$$\therefore b < -5$$

 \therefore III is NOT true.

$$\therefore -3 < a < 0 \text{ and } b < -5$$

$$\therefore a > b$$

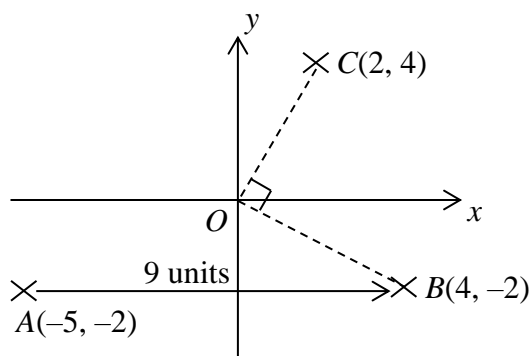
 \therefore I is true.

24. A

Rewrite the equations of the straight lines as $y = -\frac{3}{2}x - \frac{k}{2}$ and $y = -\frac{k}{12}x + \frac{1}{2}$.

$$\therefore -\frac{3}{2} \times -\frac{k}{12} = -1$$

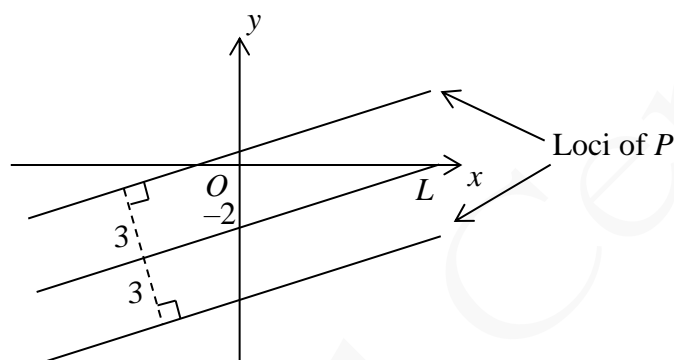
$$k = -8$$



26. D

Rewrite the equation of L as

$$y = \frac{5}{7}x - 2.$$



27. B

Rewrite the equation of the circle C as $x^2 + y^2 + 2x - 6y + \frac{15}{2} = 0$.

Centre of $C = (-1, 3)$

\therefore III is NOT true.

$$\begin{aligned} \text{Radius of } C, r &= \sqrt{(-1)^2 + (3)^2 - \left(\frac{15}{2}\right)} \\ &= \frac{\sqrt{10}}{2} \end{aligned}$$

The area of $C = \pi r^2$

$$\begin{aligned} &= \pi \left(\frac{\sqrt{10}}{2}\right)^2 \\ &= \frac{5\pi}{2} \end{aligned}$$

\therefore I is NOT true.

Distance between the centre $[(-1, 3)]$ and $(-3, 3)$

$$= 2 > \frac{\sqrt{10}}{2}$$

i.e. $(-3, 3)$ lies outside C .

\therefore II is true.

28. C

Favourable outcomes include (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8) and (8, 9). There are 8 favourable outcomes.

Number of possible outcomes = C_2^9

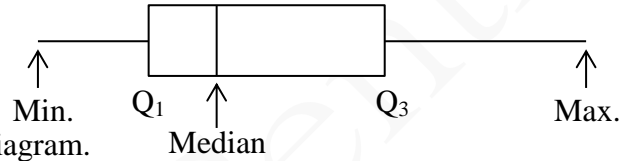
$$\begin{aligned} \therefore \text{The required probability} &= \frac{8}{C_2^9} \\ &= \frac{2}{9} \end{aligned}$$

29. B

Range = Maximum value – minimum value

Inter-quartile range = $Q_3 - Q_1$

\therefore I and III can be obtained from a box-and-whisker diagram.



30. C

There are 116 students.

Mode = median = 7

The lower quartile = 6

The upper quartile = 8

31. B

$$\text{Slope} = \frac{7-0}{0-8} = -\frac{7}{8}$$

$$\log_9 y = -\frac{7}{8} \log_3 x + 7 \quad [\text{c.f. } y = mx + c]$$

$$= \log_3 x^{-\frac{7}{8}} + \log_3 3^7 \quad [\text{use } x \log y = \log y^x]$$

$$= \log_3 (3^7 \cdot x^{-\frac{7}{8}}) \quad [\text{use } \log x + \log y = \log (xy)]$$

$$\frac{\log y}{\log 9} = \frac{\log(3^7 \cdot x^{-\frac{7}{8}})}{\log 3} \quad [\text{use change of base formula}]$$

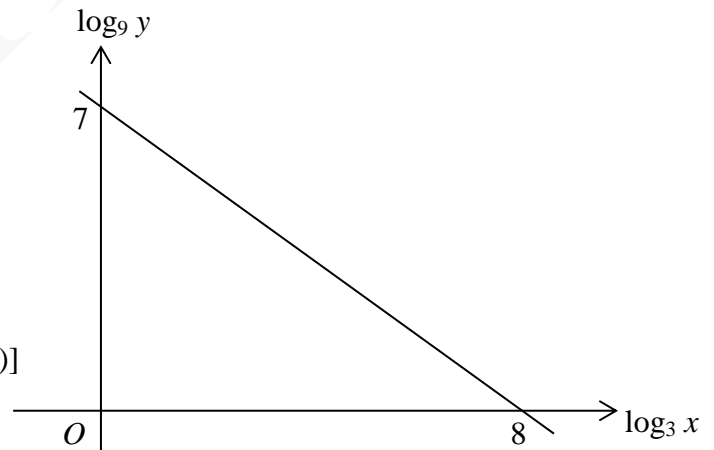
$$\frac{\log y}{2 \log 3} = \frac{\log(3^7 \cdot x^{-\frac{7}{8}})}{\log 3}$$

$$\log y^{\frac{1}{2}} = \log (3^7 \cdot x^{-\frac{7}{8}})$$

$$y^{\frac{1}{2}} = 3^7 \cdot x^{-\frac{7}{8}}$$

$$x^{\frac{7}{8}} y^{\frac{1}{2}} = 3^7$$

$$\left(x^{\frac{7}{8}} y^{\frac{1}{2}}\right)^8 = (3^7)^8 \quad \text{i.e. } x^7 y^4 = 3^{56}$$



32. D

$$\frac{3}{3 \log x - 2} + 7 = \frac{2}{2 \log x + 1}$$

$$3(2 \log x + 1) + 7(3 \log x - 2)(2 \log x + 1) = 2(3 \log x - 2)$$

$$6 \log x + 3 + 42(\log x)^2 - 7 \log x - 14 = 6 \log x - 4$$

$$6(\log x)^2 - \log x - 1 = 0$$

$$(2 \log x - 1)(3 \log x + 1) = 0 \text{ [By letting } u = \log x, \text{ this is like } (2u - 1)(3u + 1) = 0]$$

$$\log x = \frac{1}{2} \text{ or } -\frac{1}{3}$$

$$\therefore \log \frac{1}{x} = \log 1 - \log x \quad [\text{use } \log \frac{x}{y} = \log x - \log y]$$

$$= -\log x$$

$$= \frac{-1}{2} \text{ or } \frac{1}{3}$$

33. A

2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	1	0	0	0	0	0	1	0	1	1	0

$$100110000010110_2$$

$$= 2^{14} + 2^{11} + 2^{10} + 2^4 + 2^2 + 2$$

$$= (2^4 + 2^1 + 1) \times 2^{10} + 16 + 4 + 2$$

$$= (16 + 2 + 1) \times 2^{10} + 22$$

$$= 19 \times 2^{10} + 22$$

34. D

$$\frac{4+i^5}{a+i} - i^6$$

$$= \frac{4+i}{a+i} - i^2 \quad [\because i^4 = 1]$$

$$= \frac{4+i}{a+i} \times \frac{a-i}{a-i} - (-1)$$

$$= \frac{4a+ai-4i-i^2}{a^2-i^2} + 1$$

$$= \frac{4a+ai-4i-(-1)}{a^2-(-1)} + 1$$

$$= \frac{4a+1+(a-4)i}{a^2+1} + 1$$

$$= \frac{4a+1+(a-4)i+a^2+1}{a^2+1}$$

$$= \frac{a^2+4a+2}{a^2+1} + \frac{a-4}{a^2+1} i \quad \text{i.e. The real part is } \frac{a^2+4a+2}{a^2+1}.$$

35. C

Draw the straight lines of $x + 2y = 20$, $7x - 6y = 20$ and $13x + 6y = 20$.

The points of intersections are $(2, -1)$, $(-4, 12)$ and $(8, 6)$.

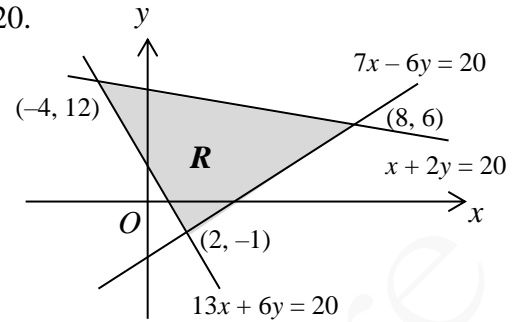
Let $P = 7x + 8y + 9$.

$$P(2, -1) = 15$$

$$P(-4, 12) = 77$$

$$P(8, 6) = 113$$

\therefore The greatest value of $7x + 8y + 9$ is 113.



36. C

Let a be the first term of the geometric sequence and r be the common ratio.

$$ar + ar^4 = 9 \quad \text{i.e.} \quad ar^6 + ar^9 = 9r^5 \quad \dots (1)$$

$$ar^6 + ar^9 = 288 \quad \dots (2)$$

Combining (1) and (2),

$$\therefore 9r^5 = 288$$

$$\text{We get } r = 2 \text{ and } a = \frac{1}{2}$$

$$\therefore 20^{\text{th}} \text{ term} = ar^{19}$$

$$= \left(\frac{1}{2}\right)(2)^{19}$$

$$= 262\,144$$

37. A

$$\begin{cases} 3x - y - 2 = 0 & \text{i.e. } y = 3x - 2 \quad \dots (1) \\ 5x^2 + 5y^2 + kx + 4y - 20 = 0 & \dots (2) \end{cases}$$

Substitute (1) into (2),

$$5x^2 + 5(3x - 2)^2 + kx + 4(3x - 2) - 20 = 0$$

$$5x^2 + 5(9x^2 - 12x + 4) + kx + 12x - 8 - 20 = 0$$

$$50x^2 + (k - 48)x - 8 = 0$$

$$\text{The } x\text{-coordinate of the mid-point of } PQ = \frac{\text{sum of roots}}{2}$$

$$= \frac{\frac{k-48}{50}}{2}$$

$$= 2$$

$$\therefore k = -152$$

38. A

$\therefore \triangle OAB$ is an equilateral \triangle .

$$\therefore \angle BOA = 60^\circ$$

$\angle OCD = \angle OAD$ (base \angle s, isos. \triangle)

$$\angle OCD + \angle OAD + \angle AOC = 180^\circ \quad (\angle \text{ sum of } \triangle)$$

$$2\angle OAD + 90^\circ = 180^\circ$$

$$\angle OAD = 45^\circ$$

$$\therefore \angle OCD = \angle OAD = 45^\circ$$

$$\begin{aligned} \angle ODC &= \angle OAD + \angle DOA \quad (\text{ext. } \angle \text{ of } \triangle) \\ &= 45^\circ + 60^\circ = 105^\circ \end{aligned}$$

By sine formula,

$$\frac{OD}{\sin \angle OCD} = \frac{OC}{\sin \angle ODC}$$

$$\frac{OD}{\sin 45^\circ} = \frac{12}{\sin 105^\circ}$$

$$\text{i.e. } OD = \frac{12 \sin 45^\circ}{\sin 105^\circ} \text{ cm}$$

$$\angle COD = 90^\circ - 60^\circ = 30^\circ$$

Area of $\triangle COD$

$$= \frac{1}{2} \times OC \times OD \times \sin \angle COD$$

$$= \frac{1}{2} \times 12 \times \frac{12 \sin 45^\circ}{\sin 105^\circ} \times \sin 30^\circ$$

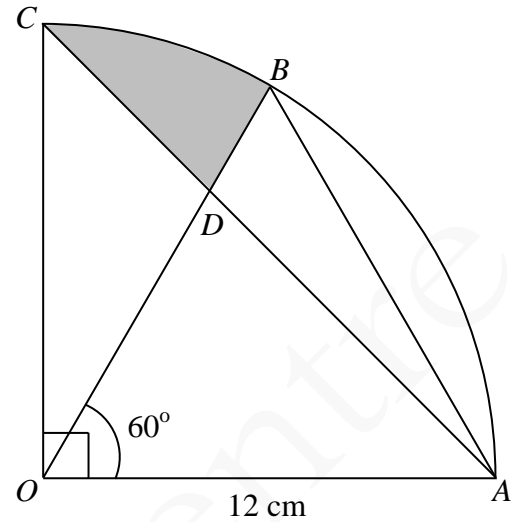
$$= \frac{36 \sin 45^\circ}{\sin 105^\circ} \text{ cm}^2$$

The area of the shaded region

$$= \text{Area of sector } BOC - \text{Area of } \triangle COD$$

$$= \pi(12)^2 \times \frac{30^\circ}{360^\circ} - \frac{36 \sin 45^\circ}{\sin 105^\circ}$$

$$= 11 \text{ cm}^2 \quad (\text{correct to the nearest cm}^2)$$



39. B

$$\angle DAE = \angle DCE \quad (\angle \text{ in the same segment})$$

$$= 22^\circ$$

$$\angle DBA = \angle DAT \quad (\angle \text{ in alt. segment})$$

$$= \angle DAE + \angle EAT$$

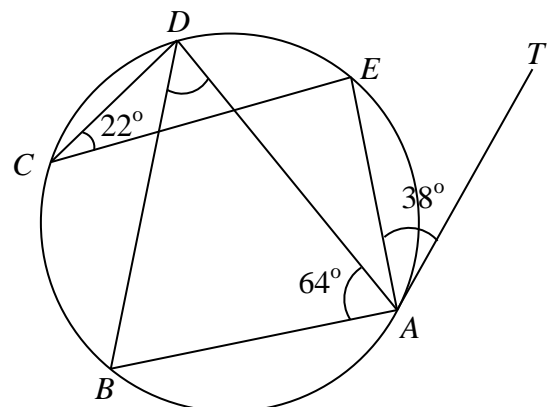
$$= 22^\circ + 38^\circ$$

$$= 60^\circ$$

$$\angle ADB + \angle DBA + \angle BAD = 180^\circ \quad (\angle \text{ sum of } \triangle)$$

$$\angle ADB + 60^\circ + 64^\circ = 180^\circ$$

$$\therefore \angle ADB = 56^\circ$$

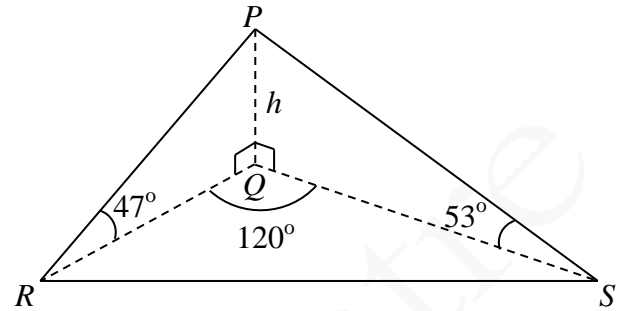


40. C

Let $PQ = h$. Then, $PR = \frac{h}{\sin 47^\circ}$, $PS = \frac{h}{\sin 53^\circ}$, $QR = \frac{h}{\tan 47^\circ}$, $QS = \frac{h}{\tan 53^\circ}$.

Consider $\triangle QRS$, by cosine formula,

$$\begin{aligned} RS^2 &= QR^2 + QS^2 - 2(QR)(QS)\cos \angle RQS \\ &= \left(\frac{h}{\tan 47^\circ}\right)^2 + \left(\frac{h}{\tan 53^\circ}\right)^2 - 2\left(\frac{h}{\tan 47^\circ}\right)\left(\frac{h}{\tan 53^\circ}\right)\cos 120^\circ \\ &= \left(\frac{h}{\tan 47^\circ}\right)^2 + \left(\frac{h}{\tan 53^\circ}\right)^2 - 2\left(\frac{h}{\tan 47^\circ}\right)\left(\frac{h}{\tan 53^\circ}\right)(-0.5) \\ &= \left(\frac{h}{\tan 47^\circ}\right)^2 + \left(\frac{h}{\tan 53^\circ}\right)^2 + \frac{h^2}{\tan 47^\circ \tan 53^\circ} \end{aligned}$$



Consider $\triangle PRS$, by cosine formula,

$$\begin{aligned} RS^2 &= PR^2 + PS^2 - 2(PR)(PS)\cos \angle RPS \\ &= \left(\frac{h}{\sin 47^\circ}\right)^2 + \left(\frac{h}{\sin 53^\circ}\right)^2 - 2\left(\frac{h}{\sin 47^\circ}\right)\left(\frac{h}{\sin 53^\circ}\right)\cos \angle RPS \\ &= \left(\frac{h}{\sin 47^\circ}\right)^2 + \left(\frac{h}{\sin 53^\circ}\right)^2 - \left(\frac{2h^2}{\sin 47^\circ \sin 53^\circ}\right)\cos \angle RPS \end{aligned}$$

$$\therefore \left(\frac{h}{\sin 47^\circ}\right)^2 + \left(\frac{h}{\sin 53^\circ}\right)^2 - \left(\frac{2h^2}{\sin 47^\circ \sin 53^\circ}\right)\cos \angle RPS = \left(\frac{h}{\tan 47^\circ}\right)^2 + \left(\frac{h}{\tan 53^\circ}\right)^2 + \frac{h^2}{\tan 47^\circ \tan 53^\circ}$$

$$\cos \angle RPS = \frac{\sin 47^\circ \sin 53^\circ}{2} \left(\frac{1}{\sin^2 47^\circ} + \frac{1}{\sin^2 53^\circ} - \frac{1}{\tan^2 47^\circ} - \frac{1}{\tan^2 53^\circ} - \frac{1}{\tan 47^\circ \tan 53^\circ} \right)$$

$\angle RPS = 68^\circ$ (correct to the nearest degree)

41. B

B is the orthocentre.

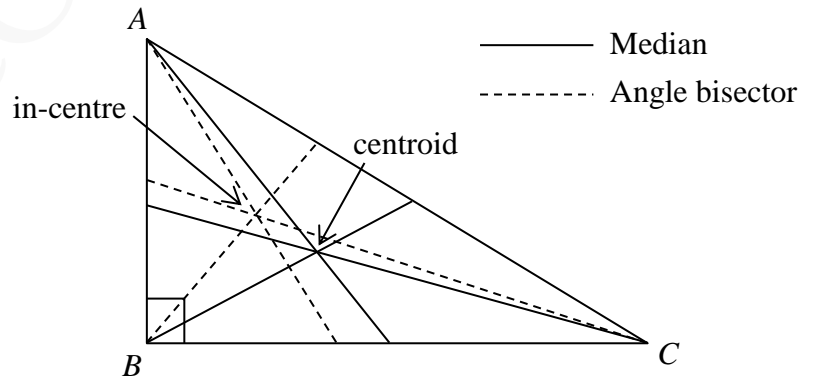
\therefore I is NOT true.

The centroid must lie inside $\triangle ABC$.

\therefore II is true.

The in-centre must lie inside $\triangle ABC$.

\therefore III is NOT true.



42. C

The required probability

$$= 1 - P(\text{"no blue cup is drawn"})$$

$$= 1 - \frac{C_6^{11}}{C_6^{19}} \quad [\text{choose from green cups and red cups}]$$

$$= \frac{635}{646}$$

43. D

The required probability = $1 - P(\text{"Susan answers all questions correctly"})$

$$\begin{aligned} &= 1 - \left(\frac{1}{3}\right) \left(\frac{1}{5}\right) \left(\frac{1}{7}\right) \\ &= \frac{104}{105} \end{aligned}$$

44. B

Let μ be the mean of the examination scores. Then,

$$\frac{69 - \mu}{8} = 0.5$$

$$\mu = 65$$

$$\begin{aligned} \therefore \text{John's examination score} \\ &= 8(-1.5) + 65 \\ &= 53 \text{ marks} \end{aligned}$$

45. A

As each number of the set is multiplied by 6 and then 5 is added to each resulting number, the new mean is also multiplied by 6 and then 5 is added.

\therefore I is true.

The new range is $6r$. Adding 5 to each number does not affect the range.

\therefore II is NOT true.

The new variance is 6^2v i.e. $36v$. Adding 5 to each number does not affect the variance.

\therefore III is NOT true.