Suggested Solution for 2020 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$\frac{6x}{(3x^{-5})^{-2}}$

$=\frac{6x}{3^{-2}x^{10}}$

$=\frac{3^2 \cdot 6}{x^9}$

$=\frac{54}{x^9}$

2. C

$a(a + b) = 2(b - a)$

$a^2 + ab = 2b - 2a$

$2b - ab = a^2 + 2a$

$b(2 - a) = a^2 + 2a$

$b = \frac{a^2+2a}{2-a}$

3. A

$\frac{5}{4k+3} - \frac{2}{4k-3}$

$=\frac{5(4k-3)-2(4k+3)}{(4k+3)(4k-3)}$

$=\frac{20k-15-8k-6}{16k^2-9}$

$=\frac{12k-21}{16k^2-9}$

4. A

$(3a + 2b)(4a - 5b) - a(6a + 4b)$

$= (3a + 2b)(4a - 5b) - 2a(3a + 2b)$

$= (3a + 2b)(4a - 5b - 2a)$

$= (3a + 2b)(2a - 5b)$

5. B

$f(1 + \beta) - f(1 - \beta)$

$= 3(1 + \beta)^2 - (1 + \beta) - 2 - [3(1 - \beta)^2 - (1 - \beta) - 2]$

$= 3(1 + 2\beta + \beta^2) - 1 - \beta - 2 - [3(1 - 2\beta + \beta^2) - 1 + \beta - 2]$

$= 3 + 6\beta + 3\beta^2 - 3 - \beta - 3 + 6\beta - 3\beta^2 + 3 - \beta$

$= 10\beta$

6. D

By Factor theorem, $g(-2) = 0$.

i.e. $a(-2)^3 + 4a(-2)^2 - 24 = 0$

$-8a + 16a - 24 = 0$

$8a - 24 = 0$

$a = 3$

$\therefore$ $g(x) = 3x^3 + 12x^2 - 24$

$g(2) = 3(2)^3 + 12(2)^2 - 24$

$= 48$

7. C

Substitute $x = -6$ into the identity,

$(-6 + h)(-6 + 6) = (-6 + 4)^2 + k$

$k + 4 = 0$

$k = -4$

<u>Alternatively</u>

L.H.S. $= x^2 + (h + 6)x + 6h$

R.H.S. $= x^2 + 8x + 16 + k$

By comparing the coefficients of x, $h + 6 = 8 \Rightarrow h = 2$

By comparing the constant terms,

$16 + k = 6h = 6(2) = 12$

$\Rightarrow$ $k = -4$

8. A

Rewrite L_1 as $y = -\frac{1}{a}x - \frac{b}{a}$ and L_2 as $y = -bx - c$.

From the figure, the y-intercept of $L_2 = -c > 0 \Rightarrow c < 0$

$\therefore$ I is true.

From the figure, the y-intercept of $L_1 = -\frac{b}{a} > 0 \Rightarrow -ab > 0$ i.e. $ab < 0 < 1$

$\therefore$ II is true.

From the figure, the slope of $L_1 = -\frac{1}{a} > 0 \Rightarrow a < 0$

Also, from the figure, the y-intercept of L_2 > the y-intercept of L_1

i.e. $-c > -\frac{b}{a}$

$\Rightarrow$ $c < \frac{b}{a}$

$\Rightarrow$ $ac > b$ $[\because a < 0]$

$\therefore$ III is NOT true.

9. A

Let C and S be the cost and the selling price of the toy respectively. Then,

C = (1 – x%)S … (1)

S = (1 + 25%)C i.e. S = 1.25C … (2)

Substitute (2) into (1),

C = (1 – x%)(1.25C)

1.25(1 – x%) = 1

x = 20

10. B

Note that 1 km^2 = 1 × 10^6 m^2 = 1 × 10^{10} cm^2

Then, 0.75 km^2 = 0.75 × 10^{10} cm^2 = 7.5 × 10^9 cm^2

Let the scale of the map be 1 : n. Note that the area of the golf course on the map and the actual area of the golf course are similar figures.

$$\left(\frac{1}{n}\right)^2 = \frac{300}{7.5\times10^9}$$

n = 5000

11. A

$w = ku^3\sqrt{v}$ where k is a constant.

$w = k(4)^3\sqrt{9}$ … (1)

$8 = k(2)^3\sqrt{4}$ … (2)

(1) ÷ (2),

$$\frac{w}{8} = \frac{k(4)^3\sqrt{9}}{k(2)^3\sqrt{9}}$$

w = 96

12. D

T(1) = 3

T(2) = 3 + [2(1) + 1] = 6

T(3) = 6 + [2(2) + 1] = 11

T(4) = 11 + [2(3) + 1] = 18

T(5) = 18 + [2(4) + 1] = 27

T(6) = 27 + [2(5) + 1] = 38

T(7) = 38 + [2(6) + 1] = 51

13. D

$5 - 4x < 9$ and $\frac{2x-3}{7} > 1$

$4x > 5 - 9$ and $2x - 3 > 7$

$4x > -4$ and $2x > 10$

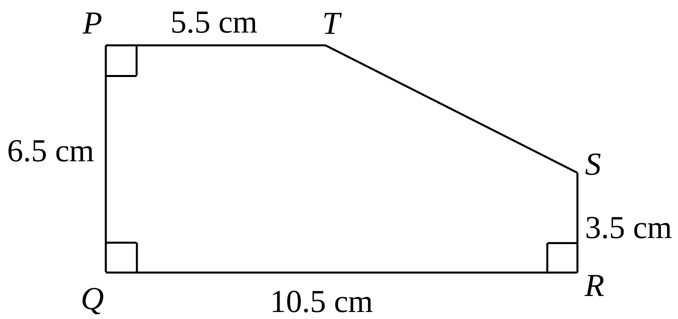
$x > -1$ and $x > 5$

$\therefore \quad x > 5$

14. B

Absolute error of the measurement = 0.5 cm

The largest possible measurements :

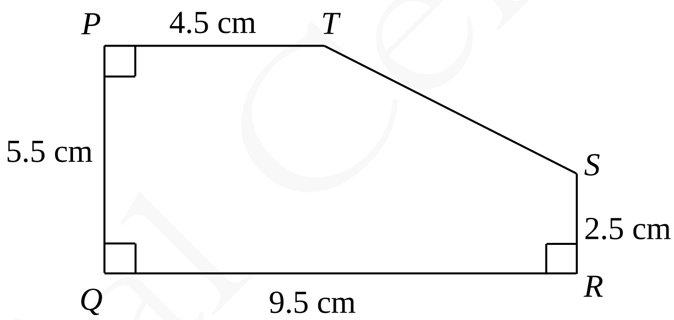


The largest possible value of A

$= 6.5 \times 5.5 + \frac{(6.5+3.5)\times(10.5-5.5)}{2}$

$= 60.75$

The smallest possible measurements :



The smallest possible value of A

$= 5.5 \times 4.5 + \frac{(5.5+2.5)\times(9.5-4.5)}{2}$

$= 44.75$

15. D

Let r and θ be the original radius and angle of the sector respectively.

$2\pi r \times \frac{\theta}{360^o} = 2\pi r(1 - 60\%) \times \frac{\theta(1+k\%)}{360^o}$

$0.4(1 + k\%) = 1$

$k = 150$

16. B

$\pi(5a)^2(7b) = 525 \quad$ i.e. $\quad \pi a^2 b = 3 \ldots (1)$

The volume of the cone

$= \frac{1}{3}\pi(7a)^2(5b)$

$= \frac{245}{3}\pi a^2 b$

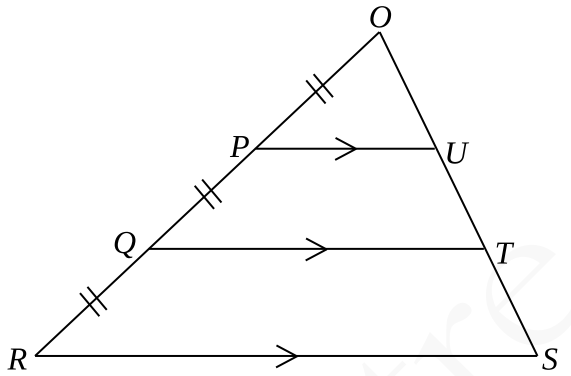
$= \frac{245}{3}(3)$ [From (1)]

$= 245 \text{ cm}^3$

17. C

Note that $\triangle OPU \sim \triangle OQT \sim \triangle ORS$.

Let the area of $\triangle OPU$ be x. Then,



$$\frac{Area\ of\ \Delta OPU}{Area\ of\ \Delta OQT} = \left(\frac{OP}{OQ}\right)^2 \text{ and } \frac{Area\ of\ \Delta OPU}{Area\ of\ \Delta ORS} = \left(\frac{OP}{OR}\right)^2$$

$$\frac{x}{Area\ of\ \Delta OQT} = \left(\frac{1}{2}\right)^2 \text{ and } \frac{x}{Area\ of\ \Delta ORS} = \left(\frac{1}{3}\right)^2$$

$\therefore$ Area of $\triangle OQT = 4x$ and area of $\triangle ORS = 9x$

Area of trapezium $PQTU$: area of trapezium $QRST$

= (Area of $\triangle OQT$ – area of $\triangle OPU$) : (area of $\triangle ORS$ – area of $\triangle OQT$)

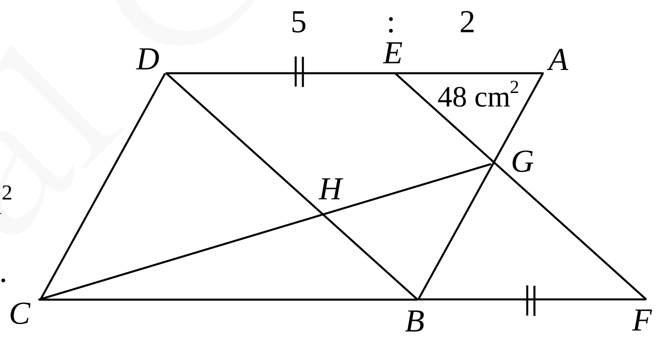
= $(4x - x) : (9x - 4x)$

= 3 : 5

18. B

Note that $EA : BC = 2 : 7$.

Note that $\triangle AEG \sim \triangle CBD$.



$$\therefore \quad \frac{Area\ of\ \Delta AEG}{Area\ of\ \Delta CBD} = \left(\frac{EA}{BC}\right)^2$$

i.e. $\frac{48}{Area\ of\ \Delta CBD} = \left(\frac{2}{7}\right)^2$ ➔ Area of $\triangle CBD = 588 \text{ cm}^2$

Note that $\triangle AEG \sim \triangle BFG$ with $AG : BG = AE : BF = 2 : 5$.

Then, $BG : DC = 5 : 7$.

Also note that $\triangle BGH \sim \triangle DCH$ with $BH : DH = BG : DC = 5 : 7$.

Area of $\triangle CBH$: area of $\triangle CDH = BH : DH = 5 : 7$ ($\because$ $\triangle CBH$ and $\triangle CDH$ have the same height.)

Let area of $\triangle CBH = 5k$ and area of $\triangle CDH = 7k$ where k is a constant.

Then, area of $\triangle CBD$ = area of $\triangle CBH$ + area of $\triangle CDH$

i.e. $588 = 5k + 7k$ ➔ $k = 49$

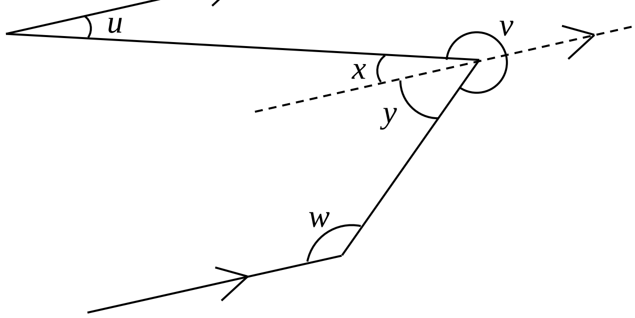
$\therefore$ Area of $\triangle CDH = 7k$

$= 7(49)$

$= 343 \text{ cm}^2$

19. B

Draw a straight line parallel to the parallel lines as shown.



$x = u$ (alt. $\angle$s, // lines)

$y + w = 180^\circ$ (int. $\angle$s, // lines)

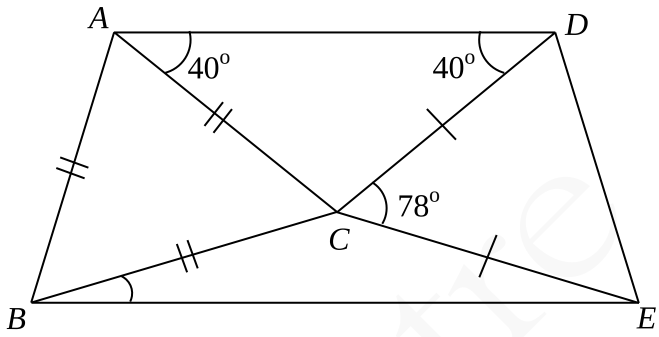
$y = 180^\circ - w$

$x + y + v = 360^\circ$ ($\angle$s at a pt.)

$u + (180^\circ - w) + v = 360^\circ$

➔ $u + v - w = 180^\circ$

20. D



$\angle ACB = 60^\circ$ (prop. of equilateral $\triangle$)

$\angle ACD + \angle CAD + \angle CDA = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle ACD + 40^\circ + 40^\circ = 180^\circ$

$\angle ACD = 100^\circ$

$\angle BCE + \angle ACB + \angle ACD + \angle DCE = 360^\circ$ ($\angle$s at a pt.)

$\angle BCE + 60^\circ + 100^\circ + 78^\circ = 360^\circ$

$\angle BCE = 122^\circ$

$CA = CD$ (sides opp. equal $\angle$s)

$\therefore \quad BC = CA = CD = CE$

$\angle CBE = \angle CEB$ (base $\angle$s, isos. $\triangle$)

$\angle CBE + \angle CEB + \angle BCE = 180^\circ$ ($\angle$ sum of $\triangle$)

$2\angle CBE + 122^\circ = 180^\circ$

$\angle CBE = 29^\circ$

21. C

$BE^2 + CE^2 = 8^2 + 15^2 = 17^2 = BC^2$

$\therefore \quad \triangle BEC$ is a right-angled $\triangle$ with $\angle BEC = 90^\circ$ (converse of Pythagoras' Theorem)

Then, area of $\triangle BEC$

$= \frac{8\times 15}{2}$

$= 60 \text{ cm}^2$

The area of rectangle $ABCD = 2 \times$ area of $\triangle BEC$

$= 2 \times 60$

$= 120 \text{ cm}^2$

22. C

$\because \quad \angle ABC = 90^\circ$

$\therefore \quad AC$ is a diameter. (Converse of $\angle$ in semi-circle)

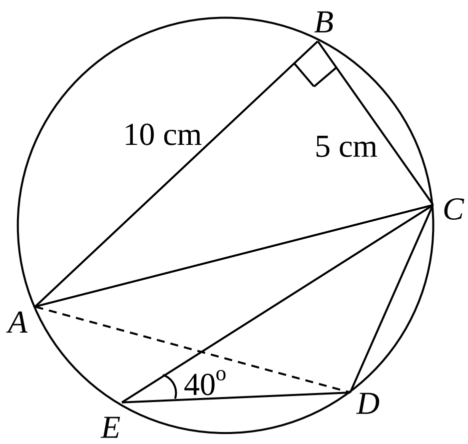
By Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= 10^2 + 5^2$

$AC = \sqrt{125}$ cm

Join AD.



$\angle ADC = 90^\circ$ ($\angle$ in semi-circle)

$\angle CAD = \angle CED$

$= 40^\circ$ ($\angle$s in the same segment)

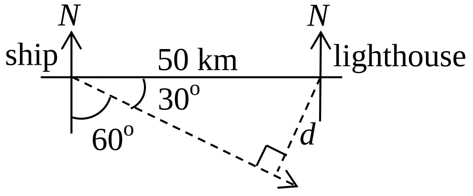
$\therefore \quad \frac{CD}{AC} = \sin 40^\circ$

$CD = \sqrt{125}\sin 40^\circ$

$= 7$ cm (correct to the nearest cm)

23. B

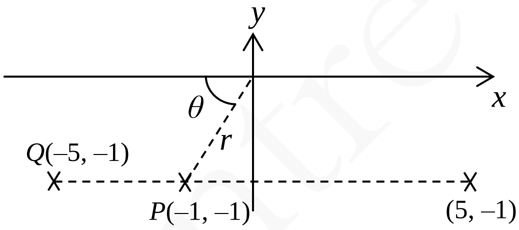
As shown, the shortest distance between the ship and the lighthouse is d. Then,



$\frac{d}{50} = \sin 30^\circ$

$d = 50 \sin 30^\circ = 25$ km

24. D



$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\tan \theta = 1$

$\theta = 45^\circ$

$\therefore$ The polar coordinates of P are $(\sqrt{2}, 180^\circ + 45^\circ)$. i.e. $(\sqrt{2}, 225^\circ)$

25. A

The locus of a moving point at a constant distance from a fixed point is a circle.

26. D

Slope of L is $-\frac{k}{4}$ and slope of the straight line $6x - 9y + 4 = 0$ is $\frac{2}{3}$.

$-\frac{k}{4} \times \frac{2}{3} = -1$ [$\because$ The two straight lines are perpendicular to each other.]

$\Rightarrow$ $k = 6$

Then, the equation of L is $6x + 4y - 12 = 0$. i.e. $3x + 2y - 6 = 0$

Substituting $x = 0$ into $3x + 2y - 6 = 0$, we get the y-intercept of L is 3.

27. C

Rewrite the equation of the circle C_1 as $x^2 + y^2 + 2x + 4y - \frac{149}{2} = 0$.

Centre of $C_1 = (-\frac{2}{2}, -\frac{4}{2})$ i.e. $(-1, -2)$

Centre of $C_2 = (-\frac{-8}{2}, -\frac{-20}{2})$ i.e. $(4, 10)$

Substitute $(-1, -2)$ into the equation of C_2, $(-1)^2 + (-2)^2 - 8(-1) - 20(-2) - 53 = 0$.

$\therefore$ I is true.

The radius of C_1, $r_1 = \sqrt{(-1)^2 + (-2)^2 - \left(-\frac{149}{2}\right)} = \sqrt{\frac{159}{2}}$

The radius of C_2, $r_2 = \sqrt{4^2 + 10^2 - (-53)} = 13 \neq r_1$

$\therefore$ II is NOT true.

Note that the centre of C_1 lies on C_2 and $r_1 < r_2$.

$\therefore$ C_1 and C_2 must intersect at two distinct points.

$\therefore$ III is true.

28. B

There are 6 possible outcomes (3, 5), (3, 7), (3, 9), (5, 7), (5, 9) and (7, 9).

Favourable outcomes are (5, 9) and (7, 9).

The required probability $= \frac{2}{6}$

$= \frac{1}{3}$

29. A

The 1st quartile, $Q_1 = 5$ and the 3rd quartile, $Q_3 = 6$

$\therefore$ The inter-quartile range $= Q_3 - Q_1$

$= 6 - 5$

$= 1$

30. A

Without loss of generality, let $m \leq n$.

If $3 \leq m \leq n \leq 8$, $x = 8$ and $z = 3$(for $m = n = 3$) or 8. The mean is not fixed.

If $3 \leq m \leq 8 \leq n \leq 12$, $x = 8$ and $z = 3$(for $m = 3$) or 8. The mean is not fixed.

If $8 \leq m \leq n \leq 12$, $x = 8$ and $z = 8$, 10($m = n = 10$) or 12($m = n = 12$). The mean is not fixed.

In all cases, $x = 8$ must be true.

31. B

$\text{B000000000000030}_{16}$

$= \text{B} \times 16^{15} + 3 \times 16^{1}$

$= 11 \times (2^4)^{15} + 48$

$= 11 \times 2^{60} + 48$

32. B

$(\log_\pi x)^2 - 10\log_\pi x + 24 = \log_\pi x$

$(\log_\pi x)^2 - 11\log_\pi x + 24 = 0$

$(\log_\pi x - 3)(\log_\pi x - 8) = 0$

$\log_\pi x = 3$ or 8

$x = \pi^3$ or π^8

$\alpha\beta = \pi^3 \times \pi^8 = \pi^{11}$

<u>Alternatively</u>

$(\log_\pi x)^2 - 11\log_\pi x + 24 = 0$

$\log_\pi \alpha + \log_\pi \beta = 11$ [$\because$ sum of roots $= -\frac{b}{a}$]

$\log_\pi(\alpha\beta) = 11$

$\alpha\beta = \pi^{11}$

33. D

As y decreases with x, $0 < a < 1$.

As y increases with x, $b > 1$.

As the graph of $y = a^x$ is the reflection image of the graph of $y = b^x$, $a = \frac{1}{b}$. i.e. $ab = 1$

34. C

Slope of the straight line = $\frac{1-0}{0-(-4)}$ = $\frac{1}{4}$. Then, the equation of the straight line is

$\sqrt{y}$ = $\frac{1}{4}x^3 + 1$

When $x = 2$,

$\sqrt{y}$ = $\frac{1}{4}(2)^3 + 1$

$\sqrt{y}$ = 3

$y = 9$

35. D

$\log a - \log a^{-3} = \log a - (-3)\log a = 4\log a$

$\log a^5 - \log a = 5\log a - \log a = 4\log a = \log a - \log a^{-3}$

$\therefore$ I is an arithmetic sequence.

$(9 - 5a) - (8 - 4a) = 9 - 5a - 8 + 4a = 1 - a$

$(10 - 6a) - (9 - 5a) = 10 - 6a - 9 + 5a = 1 - a = (9 - 5a) - (8 - 4a)$

$\therefore$ II is an arithmetic sequence.

$\cos 90^o - \cos(90 - a)^o = -\cos(90 - a)^o = -\sin a^o$

$\cos(90 + a)^o - \cos 90^o = \cos(90 + a)^o = -\sin a^o = \cos 90^o - \cos(90 - a)^o$

$\therefore$ III is an arithmetic sequence.

36. C

Draw the straight lines of $x = 2$, $2x + y + 3 = 0$ and $x + y + 1 = 0$ respectively.

Shade the region D.

The points of intersections are (0, –1), (0, –3), (2, –3), (2, –7).

Let $P = 4x + 3y + k$.

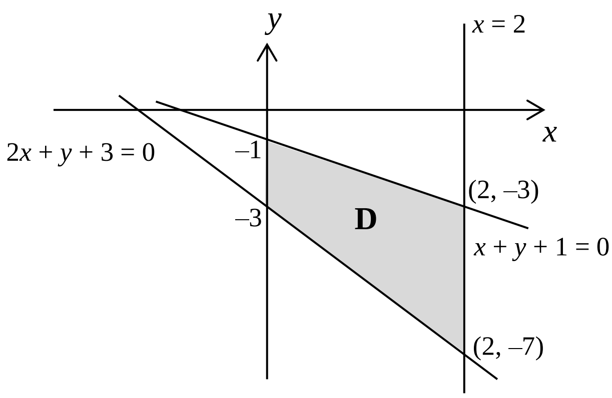
$P(0, -1) = 4(0) + 3(-1) + k = -3 + k$

$P(0, -3) = 4(0) + 3(-3) + k = -9 + k$

$P(2, -3) = 4(2) + 3(-3) + k = -1 + k$

$P(2, -7) = 4(2) + 3(-7) + k = -13 + k$

$\therefore$ The lease value $= -13 + k = 24 \Rightarrow k = 37$



37. C

$z_1 = \frac{2+ki}{1+i} \times \frac{1-i}{1-i}$

$= \frac{2+ki-2i-ki^2}{1^2-i^2}$

$= \frac{k+2}{2} + \frac{k-2}{2}i$

$z_2 = \frac{k+5i}{2-i} \times \frac{2+i}{2+i}$

$= \frac{2k+10i+ki+5i^2}{2^2-i^2}$

$= \frac{2k-5}{5} + \frac{k+10}{5}i$

$\therefore \quad \frac{k-2}{2} = \frac{k+10}{5}$

$k = 10$

$z_1 - z_2 = \frac{10+2}{2} + \frac{10-2}{2}i - (\frac{2(10)-5}{5} + \frac{10+10}{5}i)$

$= 3$

38. A

Note that $ED = AC = 12$ cm.

By Pythagoras' Theorem,

$PD^2 = EP^2 + ED^2$

$= 5^2 + 12^2$

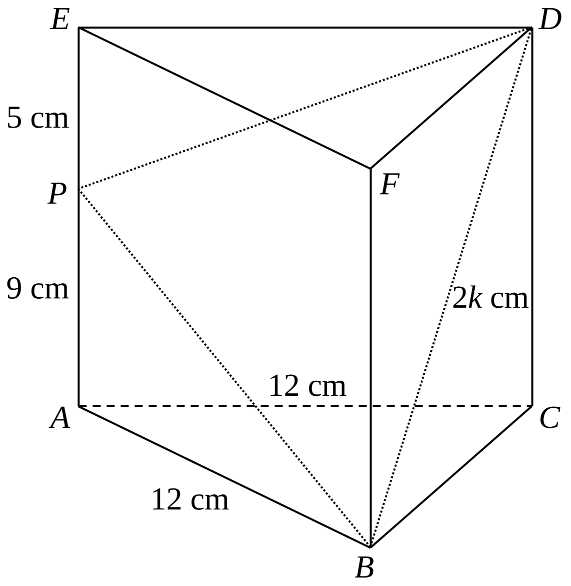
$PD = 13$ cm

By Pythagoras' Theorem,

$PB^2 = AP^2 + AB^2$

$= 9^2 + 12^2$

$PB = 15$ cm



$s = \frac{PB+PD+BD}{2}$

$= \frac{13+15+2k}{2}$

$= 14 + k$

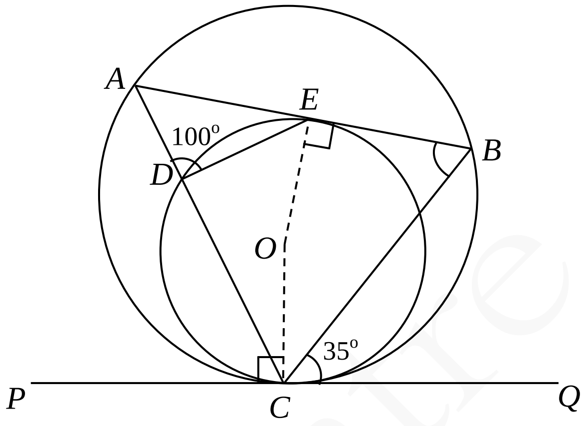
By Heron's formula,

the area of $\triangle BDP = \sqrt{(14+k)(14+k-2k)(14+k-13)(14+k-15)}$

$= \sqrt{(14+k)(14-k)(k+1)(k-1)}$

$= \sqrt{(k^2-1)(196-k^2)}$

39. A



Let O be the centre of circle CDE.

Then, $\angle OEB = \angle OCQ = 90^o$ (tangent $\perp$ radius)

$\angle OCB = \angle OCQ - \angle BCQ$

$= 90^o - 35^o$

$= 55^o$

$\angle EDC + \angle ADE = 180^o$ (adj. $\angle$s on st. line)

$\angle EDC + 100^o = 180^o$

$\angle EDC = 80^o$

$\angle EOC = 2\angle EDC$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= 2(80^o)$

$= 160^o$

$\angle ABC + \angle OEB + \angle OCB + \angle EOC = 360^o$ ($\angle$ sum of polygon)

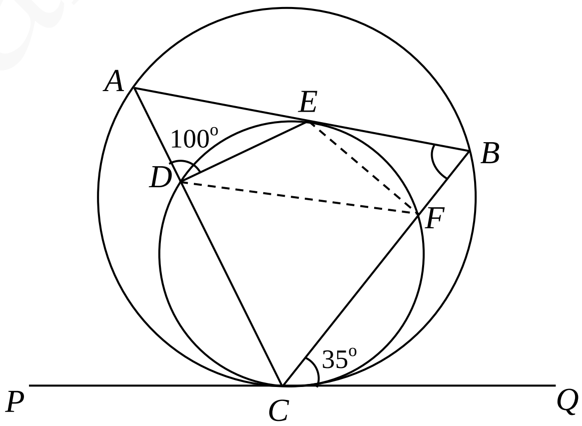
$\angle ABC + 90^o + 55^o + 160^o = 360^o$

$\angle ABC = 55^o$

<u>Alternatively</u>

Let F be the point of intersection of BC and circle CDE.

Join DF and EF.



$\angle BAC = \angle BCQ = 35^o$ ($\angle$ in alt. segment)

$\angle DEA + \angle DAE + \angle ADE = 180^o$ ($\angle$ sum of $\triangle$)

$\angle DEA + 35^o + 100^o = 180^o$

$\angle DEA = 45^o$

$\angle EFD = \angle DEA = 45^o$ ($\angle$ in alt. segment)

$\angle CDF = \angle BCQ = 35^o$ ($\angle$ in alt. segment)

$\angle EDF + \angle CDF + \angle ADE = 180^o$ (adj. $\angle$s on st. line)

$\angle EDF + 35^o + 100^o = 180^o$

$\angle EDF = 45^o$

$\angle DEF + \angle EFD + \angle EDF = 180^o$ ($\angle$ sum of $\triangle$)

$\angle DEF + 45^o + 45^o = 180^o$

$\angle DEF = 90^o$

$\angle ACB + \angle DEF = 180^o$ (opp. $\angle$s, cyclic quad.)

$\angle ACB + 90^o = 180^o$

$\angle ACB = 90^o$

$\angle ABC + \angle BAC + \angle ACB = 180^o$ ($\angle$ sum of $\triangle$)

$\angle ABC + 35^o + 90^o = 180^o$

$\angle ABC = 55^o$

40. D

As shown in the figure, the radius of the inscribed circle = $a - 31$.

The coordinates of the centre of the inscribed circle are (31, 0).

By Pythagoras' Theorem,

$AB^2 = OA^2 + OB^2$

$= 8^2 + 6^2$

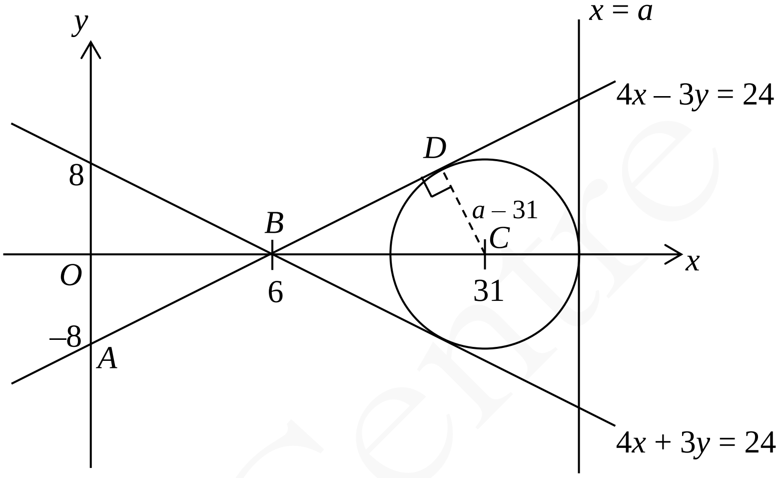
$AB = 10$

Note that $\triangle AOB \sim \triangle CDB$. Then,

$$\frac{CD}{AO} = \frac{CB}{AB}$$

$$\frac{a-31}{8} = \frac{31-6}{10}$$

$a = 51$



41. D

$$\begin{cases} x - y + 9 = 0 \quad \text{i.e. } x = y - 9 \ \ldots (1) \\ x^2 + y^2 - 6x + cy - 7 = 0 \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(y-9)^2 + y^2 - 6(y-9) + cy - 7 = 0$

$y^2 - 18y + 81 + y^2 - 6y + 54 + cy - 7 = 0$

$2y^2 + (c - 24)y + 128 = 0$

For the circle and the straight line intersect, $\Delta \geq 0$.

i.e. $(c-24)^2 - 4(2)(128) \geq 0$

$c^2 - 48c - 448 \geq 0$

$(c+8)(c-56) \geq 0$

$c \leq -8$ or $c \geq 56$

42. A

The children must queue in this way:

B_1 G_1 B_2 G_2 B_3 G_3 B_4 G_4 B_5 G_5 B_6

Number of queues = $P_6^6 \times P_5^5$

$= 86\,400$

43. B

The required probability

= 1 – P("4 Chinese books are chosen") – P("5 Chinese books are chosen")

$$= 1 - \frac{C_4^8 C_1^7}{C_5^{15}} - \frac{C_5^8}{C_5^{15}}$$

$$= \frac{9}{11}$$

44. A

Let the test scores of the students be x_1 and x_2 (where $x_1 > x_2$) respectively. Let μ and σ be the mean and standard deviation of the test scores.

$x_1 - x_2 = 30$ … (1)

$$\frac{x_1-\mu}{\sigma} - \frac{x_2-\mu}{\sigma} = 6$$

$$\frac{x_1-x_2}{\sigma} = 6 \text{ … (2)}$$

Substitute (1) into (2),

$$\frac{30}{\sigma} = 6$$

$\sigma = 5$

45. C

The question is equivalent to finding the variance of 3, 5, 9, 11, 15 and 17 as the common term $20a$ has no effect on the variance.

Mean of 3, 5, 9, 11, 15 and 17 = $\frac{3+5+9+11+15+17}{6}$ = 10

Variance = $\frac{(3-10)^2+(5-10)^2+(9-10)^2+(11-10)^2+(15-10)^2+(17-10)^2}{6}$

= 25