

**Suggested Solution for 2021 HKDSE Mathematics(core) Multiple Choice Questions**

1. B

$$\begin{aligned} & \frac{(2^n)(8^{3n})}{64^n} \\ &= \frac{(2^n)(2^3)^{3n}}{(2^6)^n} \\ &= \frac{(2^n)(2^{9n})}{2^{6n}} \\ &= 2^{n+9n-6n} \\ &= 2^{4n} \\ &= (2^2)^{2n} \\ &= 4^{2n} \end{aligned}$$

2. C

$$\begin{aligned} m(m-a) &= a(1-m) \\ m^2 - am &= a - am \\ a &= m^2 \end{aligned}$$

3. D

$$\begin{aligned} & (u+v)(u-v)(u-1) \\ &= (u^2 - v^2)(u-1) \\ &= u^2(u-1) - v^2(u-1) \\ &= u^3 - u^2 - uv^2 + v^2 \end{aligned}$$

4. B

$$\begin{aligned} & \frac{6}{n-6} - \frac{7}{n-7} \\ &= \frac{6(n-7) - 7(n-6)}{(n-6)(n-7)} \\ &= \frac{6n - 42 - 7n + 42}{(n-6)(n-7)} \\ &= \frac{-n}{(n-6)(n-7)} \\ &= \frac{n}{(n-6)(7-n)} \end{aligned}$$

5. D

$$\text{Maximum absolute error} = 0.01 \div 2 = 0.005$$

$$\text{Lower limit of } x = 6.24 - 0.005 = 6.235$$

$$\text{Upper limit of } x = 6.24 + 0.005 = 6.245$$

$$\therefore 6.235 \leq x < 6.245$$

6. D

$$a(x + 3) + b(3x + 1) \equiv c(x + 2)$$

$$(a + 3b)x + 3a + b \equiv cx + 2c$$

$$\therefore \frac{a+3b}{3a+b} = \frac{c}{2c}$$

$$2(a + 3b) = 3a + b$$

$$a = 5b$$

$$\frac{a}{b} = 5 \quad \text{i.e.} \quad a : b = 5 : 1$$

7. A

$$f(0) = 1 \rightarrow (0 + h)(0 - 3) + k = 1$$

$$-3h + k = 1 \dots (1)$$

$$f(8) = 1 \rightarrow (8 + h)(8 - 3) + k = 1$$

$$5h + k = -39 \dots (2)$$

$$(2) - (1),$$

$$8h = -40$$

$$h = -5 \dots (3)$$

Substitute (3) into (1),

$$-3(-5) + k = 1$$

$$k = -14$$

8. B

Let  $p(x) = (x^2 - 1)Q(x) + ax + b$  where  $a$  and  $b$  are constants. Note that  $ax + b$  is the remainder required.

By the Remainder theorem,

$$p(-1) = -a + b = -2 \dots (1)$$

$$p(1) = a + b = 0 \dots (2)$$

$$(1) + (2),$$

$$2b = -2$$

$$b = -1 \dots (3)$$

Substitute (3) into (2),

$$a + (-1) = 0$$

$$a = 1$$

$\therefore$  The required remainder is  $x - 1$ .

9. A

Let the number of students in the school be  $n$ . Then,

$$n \times 33\% = n \times 60\% \times 45\% + n \times (1 - 60\%) \times x\%$$

$$x = 15$$

10. C

$$9x + 8 \leq 4(x - 3) \text{ or } 6 - 7x > 20$$

$$9x + 8 \leq 4x - 12 \text{ or } -7x > 20 - 6$$

$$5x \leq -20 \text{ or } 7x < -14$$

$$x \leq -4 \text{ or } x < -2$$

$$\text{i.e. } x < -2$$

11. C

$$\frac{2\alpha + 3\beta}{3\alpha + 2\beta} = \frac{7}{10}$$

$$10(2\alpha + 3\beta) = 7(3\alpha + 2\beta)$$

$$20\alpha + 30\beta = 21\alpha + 14\beta$$

$$\alpha = 16\beta$$

$$\therefore \frac{2\alpha + \beta}{\alpha + 2\beta} = \frac{2(16\beta) + \beta}{16\beta + 2\beta}$$

$$= \frac{11}{6}$$

12. B

Let  $w = \frac{kx^2}{y^3}$  where  $k$  is a constant. Then,

$$k = \frac{wy^3}{x^2}$$

$$\therefore \frac{x^2}{wy^3} = \frac{1}{k} \text{ is also a constant.}$$

13. C

$$T_1 = 3$$

$$T_2 = 3 + 2(1) + 3 = 8$$

$$T_3 = 8 + 2(2) + 3 = 15$$

$$T_4 = 15 + 2(3) + 3 = 24$$

$$T_5 = 24 + 2(4) + 3 = 35$$

$$T_6 = 35 + 2(5) + 3 = 48$$

$$T_7 = 48 + 2(6) + 3 = 53$$

$$T_8 = 53 + 2(7) + 3 = 80$$

14. A

$$y = (m - x)^2 + n$$

$$= x^2 - 2mx + m^2 + n$$

$$\therefore a = 1 > 0$$

$\therefore$  The graph opens upwards. i.e. I is true.

$$y\text{-intercept} = m^2 + n \text{ is negative if } n < -m^2$$

$\therefore$  II may not be true.

Substitute  $x = n$  into the function,

$$y = n^2 - 2mn + m^2 + n \neq m$$

$\therefore$  III may not be true.

15. C

As shown on the right, a regular 6-sided polygon is consisted of 6 equilateral triangles.

Area of the regular 6-sided polygon

$$= \frac{1}{2} \times 8 \times 8 \times \sin 60^\circ \times 6$$

$$= 96\sqrt{3} \text{ cm}^2$$

Let  $h$  cm be the height of the prism.

$$96\sqrt{3} \times h = 288$$

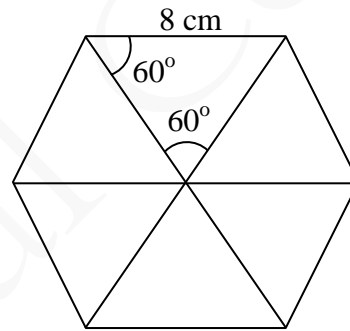
$$h = \sqrt{3}$$

Total surface area

$$= 8 \times 6 \times \sqrt{3} + 96\sqrt{3} \times 2$$

$$= 240\sqrt{3}$$

$$\approx 416 \text{ (correct to the nearest cm}^2\text{)}$$



16. A

Let  $2r$  cm and  $3r$  cm be the radii of the smaller hemisphere and the larger hemisphere respectively. Then,

$$3\pi(2r)^2 + 3\pi(3r)^2 = 351\pi$$

$$r = 3$$

$\therefore$  The radii of the two hemispheres are 6 cm and 9 cm respectively.

The difference of the volumes of the two hemispheres

$$= \frac{1}{2} \times \frac{4}{3} \times \pi(9)^3 - \frac{1}{2} \times \frac{4}{3} \times \pi(6)^3$$

$$= 342\pi \text{ cm}^3$$

17. B

Let  $r$  cm be the radius of the sector.

$$\frac{1}{4}\pi r^2 = \pi$$

$$r = 2$$

$\therefore$  I is true.

The perimeter of the sector

$$= 2r + \frac{1}{4} \times 2\pi r$$

$$= 2(2) + \frac{1}{4} \times 2\pi(2)$$

$$= 4 + \pi \text{ cm}$$

$\therefore$  II is not true.

$$\therefore \angle AOB = 90^\circ$$

$\therefore$  For the circle passing through  $O$ ,  $A$  and  $B$ ,  $AB$  is a diameter. (Converse of  $\angle$  in semi-circle)

By Pythagoras' theorem,

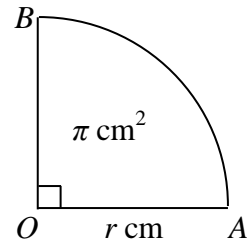
$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ &= 2^2 + 2^2 \end{aligned}$$

$$AB = 2\sqrt{2}$$

Radius of the circle passing through  $O$ ,  $A$  and  $B = AB \div 2 = \sqrt{2}$  cm

$$\text{Area of the circle} = \pi(\sqrt{2})^2 = 2\pi \text{ cm}^2$$

$\therefore$  III is true.



18. B

$$\angle BAE = \angle ADC = 28^\circ \text{ (alt. } \angle \text{s, } AB \parallel CD)$$

$$\angle ABE + \angle BAE + \angle AEB = 180^\circ \text{ (} \angle \text{ sum of } \triangle)$$

$$\angle ABE + 28^\circ + 94^\circ = 180^\circ$$

$$\angle ABE = 58^\circ$$

$$\angle BAC = \angle BCA \text{ (base } \angle \text{s, isos. } \triangle)$$

$$\angle ABC + \angle BCA + \angle BAC = 180^\circ \text{ (} \angle \text{ sum of } \triangle)$$

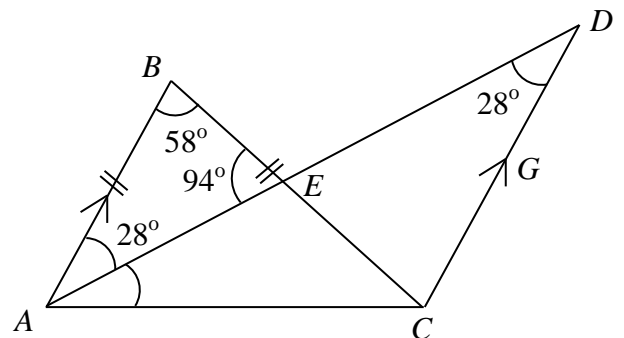
$$58^\circ + 2\angle BAC = 180^\circ$$

$$\angle BAC = 61^\circ$$

$$\angle CAD = \angle BAC - \angle BAE$$

$$= 61^\circ - 28^\circ$$

$$= 33^\circ$$



19. D

Let  $\angle DAE = x$ . Then,  $\angle ACB = \angle EGC = \angle ABE = x$ .

Also,  $\angle CAB = \angle ECG = \angle CBE = 90^\circ - x$

$\angle CFE = \angle CBE = 90^\circ - x$  (base  $\angle$ s, isos.  $\Delta$ )

$\angle DGF = \angle EGC = x$  (vert. opp.  $\angle$ s)

i.e.  $\angle DAE = \angle DGF$

$\therefore$  I is true.

In  $\Delta BCE$  and  $\Delta CGE$ ,

$\angle BCE = \angle CGE = x$

$\angle CBE = \angle GCE = 90^\circ - x$

$\angle CEB = \angle GEC = 90^\circ$

$\therefore \Delta BCE \sim \Delta CGE$  (AAA)

$\therefore$  II is true.

In  $\Delta BCE$  and  $\Delta FCE$ ,

Note that  $BC = AD$  (Properties of rectangle) and  $CF = AD$  (Given)

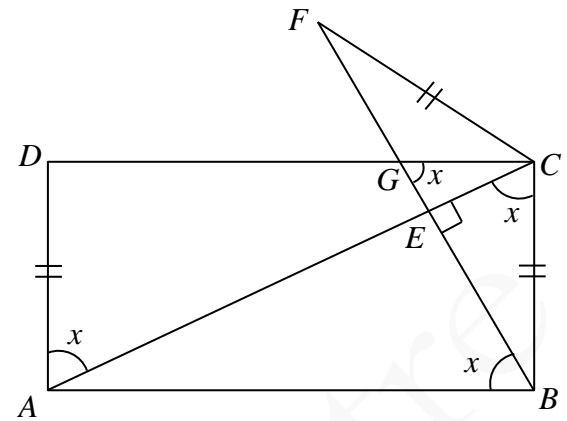
$\therefore BC = FC$

$CE = CE$  (common)

$\angle CEB = \angle CEF = 90^\circ$

$\therefore \Delta BCE \cong \Delta FCE$  (RHS)

$\therefore$  III is true.



20. C

Let  $BE = x$ . Then,  $AE = 3x$  and  $AD = 4x$ .

By Pythagoras' Theorem,

$$DE^2 = AE^2 + AD^2 \\ = (3x)^2 + (4x)^2$$

$$DE = 5x$$

Note that  $\Delta DAE \sim \Delta EBF$ .

$$\frac{EF}{DE} = \frac{EB}{DA} \quad \text{and} \quad \frac{BF}{AE} = \frac{EB}{DA}$$

$$\frac{EF}{5x} = \frac{x}{4x} \quad \text{and} \quad \frac{BF}{3x} = \frac{x}{4x}$$

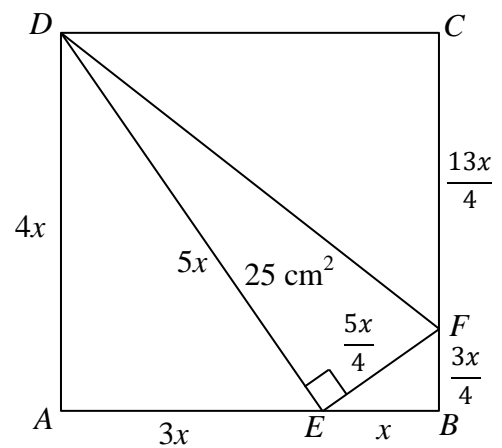
$$EF = \frac{5x}{4} \quad \text{and} \quad BF = \frac{3x}{4}$$

$$\text{Also, } CF = BC - BF = 4x - \frac{3x}{4} = \frac{13x}{4}$$

$$\text{Area of } \Delta DEF = \frac{1}{2} \times 5x \times \frac{5x}{4} = 25$$

$$\rightarrow x^2 = 8$$

$$\text{Area of } \Delta CDF = \frac{1}{2} \times 4x \times \frac{13x}{4} = \frac{13x^2}{2} = 52 \text{ cm}^2$$



21. D

$$\text{Each interior angle} = \frac{(8-2) \times 180^\circ}{8} = 135^\circ$$

Note that  $\triangle HAG$  is an isosceles  $\triangle$ .

$$\angle HAG = \angle HGA \text{ (base } \angle \text{s, isos. } \triangle)$$

$$\angle AHG + \angle HAG + \angle HGA = 180^\circ \text{ (} \angle \text{ sum of } \triangle)$$

$$135^\circ + 2 \angle HAG = 180^\circ$$

$$\angle HAG = 22.5^\circ$$

$$\angle BAG = \angle HAB - \angle HAG = 135^\circ - 22.5^\circ = 112.5^\circ$$

$$\text{By symmetry, } \angle ABF = \angle CBF = \frac{135^\circ}{2} = 67.5^\circ$$

$$\angle BAG + \angle ABF = 112.5^\circ + 67.5^\circ = 180^\circ$$

$$\therefore AG \parallel BF$$

$\therefore$  I is true.

It is not difficult to see that  $\triangle BCD \cong \triangle EFG$ .

$$\therefore BD = EG \text{ (corr. sides, } \cong \triangle \text{s)}$$

$\therefore$  II is true.

By symmetry,  $\angle BAC = \angle HAG = 22.5^\circ$ .

$$\angle CAG = \angle BAG - \angle BAC$$

$$= 112.5^\circ - 22.5^\circ$$

$$= 90^\circ$$

$$\angle CDH = \angle CDH \div 2 = 135^\circ \div 2 = 67.5^\circ$$

Note that  $\angle CDB = \angle HAG = 22.5^\circ$ .

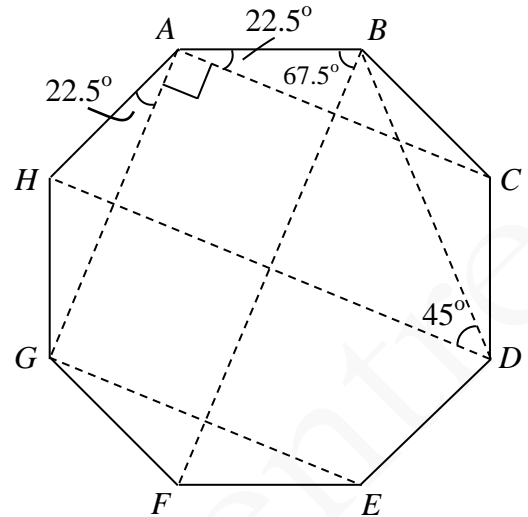
$$\therefore \angle BDH = \angle CDH - \angle CDB$$

$$= 67.5^\circ - 22.5^\circ$$

$$= 45^\circ$$

$$\text{i.e. } \angle CAG = 2 \angle BDH$$

$\therefore$  III is true.



22. A

$$\angle BAC = \angle BDC = 14^\circ \text{ (} \angle \text{s in the same segment)}$$

$$\angle ACD + \angle AED = 180^\circ \text{ (opp. } \angle \text{s, cyclic quad.)}$$

$$\angle ACD + 96^\circ = 180^\circ \rightarrow \angle ACD = 84^\circ$$

$$\therefore AC = BD$$

$$\therefore \angle ADC = \angle DAB$$

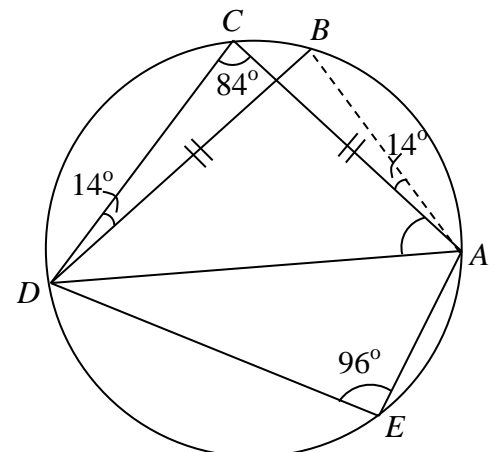
$$\text{i.e. } \angle ADB + 14^\circ = \angle CAD + 14^\circ$$

$$\angle ADB = \angle CAD$$

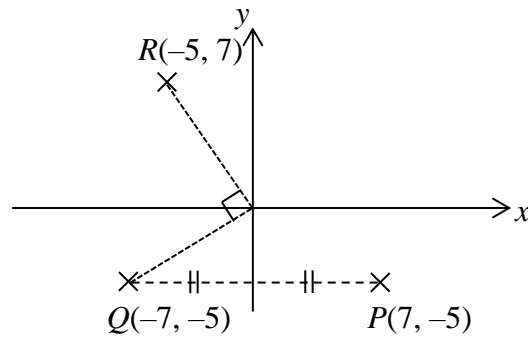
$$\angle CAD + \angle ADB + \angle BDC + \angle ACD = 180^\circ \text{ (} \angle \text{ sum of } \triangle)$$

$$2 \angle CAD + 14^\circ + 84^\circ = 180^\circ$$

$$\angle CAD = 41^\circ$$



23. B



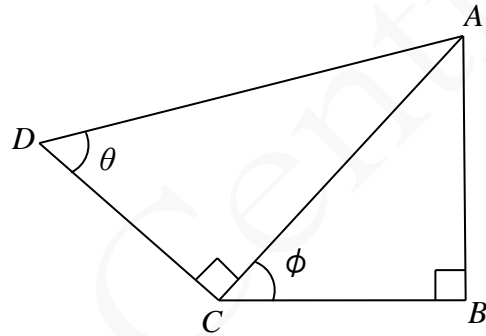
24. D

$$\sin \phi = \frac{AB}{AC} \rightarrow AC = \frac{AB}{\sin \phi}$$

$$\tan \theta = \frac{AC}{CD} \rightarrow AC = CD \tan \theta$$

$$\therefore \frac{AB}{\sin \phi} = CD \tan \theta$$

$$\text{i.e. } \frac{AB}{CD} = \tan \theta \sin \phi$$



25. C

Let the coordinates of  $P$  be  $(x, y)$ .

Then the equation of the locus of  $P$  is

$$\sqrt{(x-5)^2 + (y-7)^2} = \sqrt{(6-5)^2 + (8-7)^2}$$

$$x^2 - 10x + 25 + y^2 - 14y + 49 = 2$$

$$x^2 + y^2 - 10x - 14y + 72 = 0$$

26. A

Note that  $P$  is the mid-point of  $B$  and  $C$ .

$$P = \left( \frac{5+9}{2}, \frac{8+2}{2} \right) = (7, 5)$$

$$\text{Slope of } AP = \frac{5-3}{7-3} = \frac{1}{2}$$

Let the equation of  $AP$  be  $y = \frac{1}{2}x + c$ .

Substitute  $(3, 3)$  or  $(7, 5)$  into the equation,

$$3 = \frac{1}{2}(3) + c \quad \text{or} \quad 5 = \frac{1}{2}(7) + c$$

$$c = \frac{3}{2}$$

$$\therefore y = \frac{1}{2}x + \frac{3}{2}$$

$$\text{i.e. } x - 2y + 3 = 0$$

Alternatively

The equation of  $AP$  is

$$\frac{y-3}{x-3} = \frac{5-3}{7-3} \quad / \quad \frac{y-5}{x-7} = \frac{5-3}{7-3}$$

$$2y - 6 = x - 3 \quad / \quad 2y - 10 = x - 7$$

$$\text{i.e. } x - 2y + 3 = 0$$



27. C

Centre of the circle,  $O = \left(-\frac{-18}{2}, -\frac{-20}{2}\right)$  i.e. (9, 10)

Let  $L_1$  be the straight line joining  $O$  and  $(s, t)$ .

Note that  $L_1$  is perpendicular to  $L$ . (line from centre to mid-pt. of chord  $\perp$  chord)

$\therefore$  Slope of  $L_1 \times$  slope of  $L = -1$

i.e. Slope of  $L_1 \times 4 = -1$

$$\text{Slope of } L_1 = -\frac{1}{4}$$

$$\therefore \frac{t-10}{s-9} = -\frac{1}{4}$$

i.e.  $s + 4t - 49 = 0$

28. D

Lower quartile = 62

$$\begin{aligned} \text{The required probability} &= \frac{20}{24} \\ &= \frac{5}{6} \end{aligned}$$

29. B

The 1<sup>st</sup> quartile,  $Q_1 = 26$  and the 3<sup>rd</sup> quartile,  $Q_3 = 36$

$$\begin{aligned} \therefore \text{The inter-quartile range} &= Q_3 - Q_1 \\ &= 36 - 26 \\ &= 10 \end{aligned}$$

30. A

$$\begin{aligned} \text{The mean of the remaining 40 integers} &= \frac{70 \times 32 - 30 \times 24}{40} \\ &= 38 \end{aligned}$$

31. A

From the H.C.F. of the expressions,  $x^2$  and  $z$  must come from the 3<sup>rd</sup> expression.

From the L.C.M. of the expression,  $y^4$  must come from the 3<sup>rd</sup> expression.

$\therefore$  The 3<sup>rd</sup> expression is  $x^2y^4z$ .

32. D

$$\begin{aligned} &14 \times 16^{15} + 17 \times 16^{14} + 16^2 + 17 \\ &= 14 \times 16^{15} + (16 + 1) \times 16^{14} + 16^2 + 16 + 1 \\ &= 15 \times 16^{15} + 1 \times 16^{14} + 1 \times 16^2 + 1 \times 16 + 1 \times 16^0 \\ &= \text{F100000000000111}_{16} \end{aligned}$$

33. C

Substitute  $y = 0$  into  $y = a + \log_b x$ ,

$$0 = a + \log_b x$$

$$x = b^{-a} = \frac{1}{b^a} \quad \text{i.e. } S = \left(\frac{1}{b^a}, 0\right)$$

Substitute  $y = 0$  into  $y = \log_c x$ ,

$$0 = \log_c x$$

$$x = 1 \quad \text{i.e. } T = (1, 0)$$

$$\begin{aligned} \therefore OT : OS &= 1 : \frac{1}{b^a} \\ &= b^a : 1 \end{aligned}$$

34. C

Slope of the straight line =  $\frac{2-0}{0-(-4)} = \frac{1}{2}$ . Then, the equation of the straight line is

$$\log_5 y = \frac{1}{2} \log_5 x + 2$$

$$= \log_5 x^{\frac{1}{2}} + \log_5 5^2$$

$$= \log_5 (25x^{\frac{1}{2}})$$

$$\therefore y = 25x^{\frac{1}{2}}$$

$$y^2 = 625x \quad \text{i.e. } \frac{y^2}{x} = 625$$

Alternatively

From the graph, we get

$$\begin{cases} \log_5 x = 0 & \text{i.e. } x = 1 \\ \log_5 y = 2 & \text{i.e. } y = 5^2 = 25 \end{cases}$$

Similarly,

$$\begin{cases} \log_5 x = -4 & \text{i.e. } x = 5^{-4} \\ \log_5 y = 0 & \text{i.e. } y = 1 \end{cases}$$

From the choices of the answer, we see that  $x^m y^n = 625$  where  $m$  and  $n$  are constants.Substitute  $(1, 25)$  into  $x^m y^n = 625$ ,

$$(1)^m (25)^n = 625$$

$$\rightarrow n = 2$$

Substitute  $(5^{-4}, 1)$  into  $x^m y^n = 625$ ,

$$(5^{-4})^m (1)^n = 625$$

$$\rightarrow m = -1$$

$$\therefore x^{-1} y^2 = 625 \quad \text{i.e. } \frac{y^2}{x} = 625$$

35. A

$$\begin{aligned}
 u &= w + \frac{1}{w} = \frac{\alpha+i}{\alpha-i} + \frac{\alpha-i}{\alpha+i} \\
 &= \frac{(\alpha+i)^2 + (\alpha-i)^2}{(\alpha-i)(\alpha+i)} \\
 &= \frac{\alpha^2 + 2\alpha i + i^2 + \alpha^2 - 2\alpha i + i^2}{\alpha^2 + 1} \\
 &= \frac{2\alpha^2 - 2}{\alpha^2 + 1}
 \end{aligned}$$

∴ I is true.

$$\begin{aligned}
 v &= w - \frac{1}{w} = \frac{\alpha+i}{\alpha-i} - \frac{\alpha-i}{\alpha+i} \\
 &= \frac{(\alpha+i)^2 - (\alpha-i)^2}{(\alpha-i)(\alpha+i)} \\
 &= \frac{\alpha^2 + 2\alpha i + i^2 - \alpha^2 + 2\alpha i - i^2}{\alpha^2 + 1} \\
 &= \frac{4\alpha i}{\alpha^2 + 1}
 \end{aligned}$$

∴ II is true.

$$\begin{aligned}
 w &= \frac{\alpha+i}{\alpha-i} \\
 &= \frac{(\alpha+i)^2}{(\alpha-i)(\alpha+i)} \\
 &= \frac{\alpha^2 - 1}{\alpha^2 + 1} + \frac{2\alpha}{\alpha^2 + 1} i
 \end{aligned}$$

$$2w = \frac{2(\alpha^2 - 1)}{\alpha^2 + 1} + \frac{4\alpha}{\alpha^2 + 1} i$$

∴ III is not true.

36. A

Let  $t$  be the common ratio. Then,  $q = pt$ ,  $r = pt^2$  and  $s = pt^3$ .

$$= p(pt^3) = p^2 t^3$$

$$qr = (pt)(pt^2) = p^2 t^3$$

$$\text{i.e. } ps = qr$$

∴ I is true.

$$p + s = p + pt^3$$

$$q + r = pt + pt^2$$

∴ II may not be true.

∴  $p < q < r < s$  only when  $t > 1$ .

∴ III may not be true.

37. B

$$\Delta \leq 0$$

$$\rightarrow k^2 - 4(1)(k + 8) \leq 0$$

$$k^2 - 4k - 32 \leq 0$$

$$(k - 8)(k + 4) \leq 0$$

$$-4 \leq k \leq 8$$

38. B

Let  $T$  be the foot of the perpendicular line from  $E$  to  $CD$ . Then,  $ET = BC = 597$  cm.

$$BE = AB - AE = 960 - 638 = 322 \text{ cm}$$

$$GT = CG - BE = 480 - 322 = 158 \text{ cm}$$

$$\tan \angle EGT = \frac{ET}{GT} = \frac{597}{158}$$

$$\angle EGT = \tan^{-1}\left(\frac{597}{158}\right) \approx 75.17615796^\circ$$

$$\begin{aligned} \angle AEG &= \angle EGT \text{ (alt. } \angle \text{s, } AB \parallel CD) \\ &\approx 75.17615796^\circ \end{aligned}$$

$$\tan \angle AFB = \frac{AB}{BF} = \frac{960}{280}$$

$$\angle AFB = \tan^{-1}\left(\frac{960}{280}\right) \approx 73.73979529^\circ$$

$$\begin{aligned} \angle HAD &= \angle AFB \text{ (alt. } \angle \text{s, } AB \parallel CD) \\ &\approx 73.73979529^\circ \end{aligned}$$

$$\angle FAB + \angle AFB + \angle ABF = 180^\circ \text{ (} \angle \text{ sum of } \Delta)$$

$$\angle FAB + 73.73979529^\circ + 90^\circ = 180^\circ$$

$$\angle FAB \approx 16.26020471^\circ$$

$$\angle AHE + \angle AEH + \angle HAE = 180^\circ \text{ (} \angle \text{ sum of } \Delta)$$

$$\angle AHE + 75.17615796^\circ + 16.26020471^\circ = 180^\circ$$

$$\angle AHE \approx 88.56363733^\circ$$

By sine formula,

$$\frac{AH}{\sin \angle AEH} = \frac{AE}{\sin \angle AHE}$$

$$\frac{AH}{\sin 75.17615796^\circ} = \frac{638}{\sin 88.56363733^\circ}$$

$$AH \approx 616.9593106 \text{ cm}$$

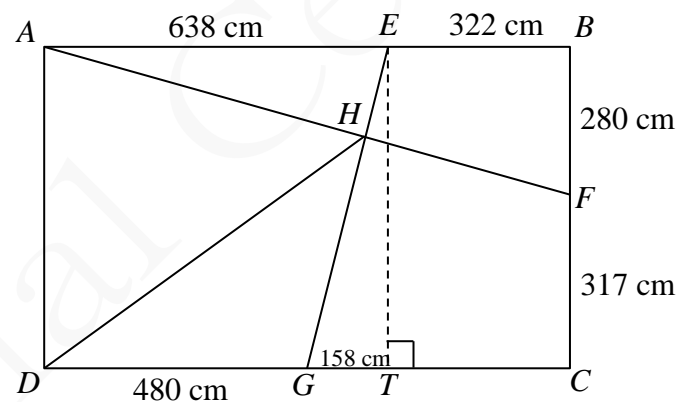
By cosine formula,

$$DH^2 = AD^2 + AH^2 - 2(AD)(AH)\cos \angle HAD$$

$$\approx 597^2 + (616.9593106)^2 + 2(597)(616.9593106)\cos 73.73979529^\circ$$

$$DH \approx 728.5505845 \text{ cm}$$

$$\approx 729 \text{ cm}$$



39. D

Join AD.

$$\angle CAD = \angle CDF = 49^\circ \text{ (}\angle \text{ in alt. segment)}$$

$$\angle CDA = 90^\circ \text{ (}\angle \text{ in semi-circle)}$$

$$\angle DCE + \angle CED = \angle CDF \text{ (ext. } \angle \text{ of } \triangle)$$

$$\angle DCE + 31^\circ = 49^\circ$$

$$\angle DCE = 18^\circ$$

$$\angle ACD + \angle CDA + \angle CAD = 180^\circ \text{ (}\angle \text{ sum of } \triangle)$$

$$(\angle ECA + \angle DCE) + \angle CDA + \angle CAD = 180^\circ$$

$$\angle ECA + 18^\circ + 90^\circ + 49^\circ = 180^\circ$$

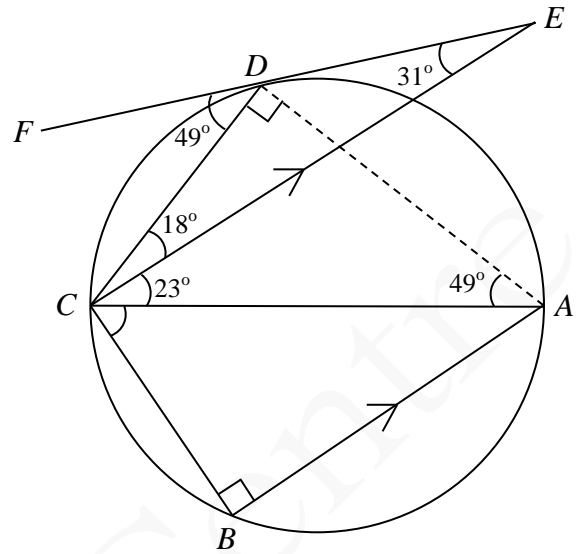
$$\angle ECA = 23^\circ$$

$$\angle ABC = 90^\circ \text{ (}\angle \text{ in semi-circle)}$$

$$\angle ACB + \angle ECA + \angle ABC = 180^\circ \text{ (int. } \angle \text{s, } AB \parallel EC)$$

$$\angle ACB + 23^\circ + 90^\circ = 180^\circ$$

$$\angle ACB = 67^\circ$$



40. A

$$\begin{cases} 4x = 3y \text{ i.e. } y = \frac{4}{3}x \dots (1) \\ x^2 + y^2 - 4x - 22y + 75 = 0 \dots (2) \end{cases}$$

Substitute (1) into (2),

$$x^2 + \left(\frac{4}{3}x\right)^2 - 4x - 22\left(\frac{4}{3}x\right) + 75 = 0$$

$$x^2 - 12x + 27 = 0$$

$$(x - 3)(x - 9) = 0$$

$$x = 3 \text{ or } 9$$

$$\text{When } x = 3, y = 4.$$

$$\text{When } x = 9, y = 12.$$

The centre of the required circle is the mid-point of (3, 4) and (9, 12). i.e.  $\left(\frac{3+9}{2}, \frac{4+12}{2}\right) = (6, 8)$

$$\text{Radius of the required circle} = \frac{\sqrt{(9-3)^2 + (12-4)^2}}{2} = 5$$

The equation of the required circle is  $(x - 6)^2 + (y - 8)^2 = 5^2$ .

Alternatively

Let  $L$  be the line joining the centre of the circle  $x^2 + y^2 - 4x - 22y + 75 = 0$  and the mid-point of  $MN$ .

Note that  $L \perp MN$  and bisects  $MN$ . Therefore, the slope of  $L \times$  the slope of  $MN = -1$

$$\text{i.e. } m_L \times \frac{4}{3} = -1 \rightarrow m_L = -\frac{3}{4}$$

$$\text{Centre of the circle } x^2 + y^2 - 4x - 22y + 75 = 0 = \left(-\frac{-4}{2}, -\frac{-22}{2}\right) = (2, 11)$$

Let the equation of  $L$  be  $y = -\frac{3}{4}x + c$ .

Substitute  $(2, 11)$  into the equation of  $L$ ,

$$11 = -\frac{3}{4}(2) + c$$

$$c = \frac{25}{2}$$

i.e. The equation of  $L$  is  $y = -\frac{3}{4}x + \frac{25}{2}$ .

$$y = \frac{4}{3}x \dots (1)$$

$$y = -\frac{3}{4}x + \frac{25}{2} \dots (2)$$

Solving (1) and (2), we get the intersection of the straight lines to be  $(6, 8)$  which is the centre of the required circle.

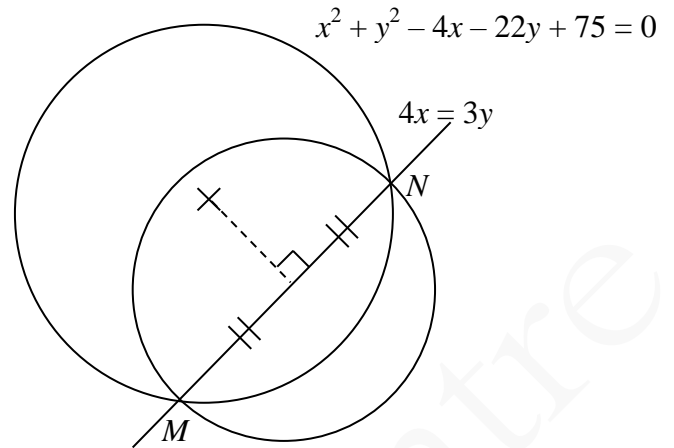
$$\text{Distance between } (6, 8) \text{ and } (2, 11) = \sqrt{(6-2)^2 + (11-8)^2} = 5$$

$$\text{Radius of the circle } x^2 + y^2 - 4x - 22y + 75 = 0 \text{ is } \sqrt{2^2 + 11^2 - 75} = \sqrt{50}$$

By Pythagoras' Theorem,

$$\text{The radius of the required circle} = \sqrt{50 - 5^2} = 5$$

$$\therefore \text{The equation of the required circle is } (x-6)^2 + (y-8)^2 = 5^2.$$



41. C

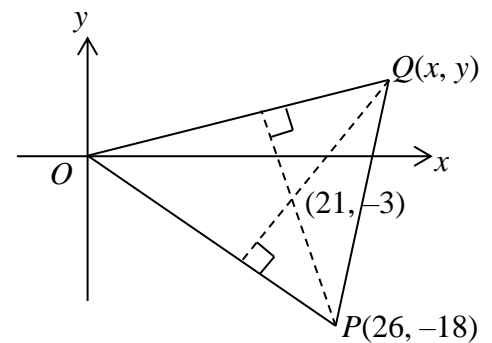
Let  $Q = (x, y)$ .

$$\text{Then, } \frac{y-(-3)}{x-21} \times \frac{-18-0}{26-0} = -1 \quad \text{i.e. } 13x - 9y - 300 = 0 \dots (1)$$

$$\text{and } \frac{y-0}{x-0} \times \frac{-18-(-3)}{26-21} = -1 \quad \text{i.e. } x - 3y = 0 \dots (2)$$

Solving (1) and (2),

$$x = 30 \text{ and } y = 10$$



42. D

$$\begin{aligned} \text{Number of choirs} &= C_4^{20} \times C_3^{10} + C_5^{20} \times C_2^{10} + C_6^{20} \times C_1^{10} + C_7^{20} \\ &= 1\,744\,200 \end{aligned}$$

43. D

The required probability

$$= \frac{5}{15} + \frac{10}{15} \times \frac{5}{14} + \frac{10}{15} \times \frac{9}{14} \times \frac{5}{13}$$

$$= \frac{67}{91}$$

44. C

Let  $\sigma$  be the standard deviation of the examination scores.

$$\frac{25-45}{\sigma} = -5$$

$$\sigma = 4$$

Let  $x$  be the girl's examination score.

$$\frac{x-45}{4} = 7$$

$$x = 73$$

45. B

Let  $d$  be the common difference of the arithmetic sequence.

For  $d < 0$ ,  $x_1 > x_2$ .

$\therefore$  I may not be true.

$$T(49) - T(1) = 48d$$

$$T(99) - T(51) = 48d = T(49) - T(1)$$

$\therefore y_1 = y_2$  i.e. II must be true.

Since  $T(n)$  is an arithmetic sequence,  $z_1 = z_2$ .

$\therefore$  III may not be true.