Suggested Solution for 2022 HKDSE Mathematics(core) Multiple Choice Questions

1. D

$\alpha^2 - \alpha - \beta^2 + \beta$

$= \alpha^2 - \beta^2 - \alpha + \beta$

$= (\alpha + \beta)(\alpha - \beta) - (\alpha - \beta)$

$= (\alpha - \beta)(\alpha + \beta - 1)$

2. B

$\frac{81^{2n+3}}{(27^{n+1})^2}$

$=\frac{(3^4)^{2n+3}}{[(3^3)^{n+1}]^2}$

$=\frac{3^{8n+12}}{(3^{3n+3})^2}$

$= \frac{3^{8n+12}}{3^{6n+6}}$

$= 3^{2n+6}$

3. A

$(x + 3)^2 + mx \equiv (x - n)(x + 1) + 2n$

$x^2 + 6x + 9 + mx \equiv x(x + 1) - n(x + 1) + 2n$

$x^2 + (6 + m)x + 9 \equiv x^2 + (1 - n)x + n$

By comparing coefficients,

$n = 9$ and $6 + m = 1 - n$

$\therefore \quad m = -14$

Alternatively

Substitute $x = 0$ into both sides of the identity.

$(0 + 3)^2 + m(0) = (0 - n)(0 + 1) + 2n$

➔ $n = 9$

Now, substitute $x = 9$ into both sides of the identity.

$(9 + 3)^2 + m(9) = (9 - 9)(9 + 1) + 2(9)$

$\therefore \quad m = -14$

4. D

$(x - c)(x - 4c) = (3c - x)(x - 4c)$

$(x - c)(x - 4c) - (3c - x)(x - 4c) = 0$

$(x - 4c)[(x - c) - (3c - x)] = 0$

$(x - 4c)(2x - 4c) = 0$

$(x - 4c)(x - 2c) = 0$

$x = 2c$ or $4c$

5. A

$\frac{2}{u} + \frac{3}{v} = 4$

Multiply both sides by uv.

$2v + 3u = 4uv$

$4uv - 3u = 2v$

$u(4v - 3) = 2v$

$u = \frac{2v}{4v-3}$

Alternatively

$\frac{2}{u} + \frac{3}{v} = 4$

$\frac{2}{u} = 4 - \frac{3}{v}$

$= \frac{4v-3}{v}$

$\therefore \quad \frac{u}{2} = \frac{v}{4v-3}$

$u = \frac{2v}{4v-3}$

6. B

For x being rounded down to 3 significant figures, it must be larger than or equal to 345 but smaller than 346.

$\therefore \quad 345 \leq x < 346$

7. A

$3y - 5 < 5y + 1$ and $5y + 1 \leq 11$

$2y > -6$ and $5y \leq 10$

$y > -3$ and $y \leq 2$

$\therefore \quad -3 < y \leq 2$

8. B

$f(k) = k^2 - k + 1$

$f(-k) = (-k)^2 - (-k) + 1 = k^2 + k + 1 \neq f(k)$

$f(1 - k) = (1 - k)^2 - (1 - k) + 1$

$= 1 - 2k + k^2 - 1 + k + 1$

$= k^2 - k + 1$

$= f(k)$

$f(k + 1) - f(1) = (k + 1)^2 - (k + 1) + 1 - [(1)^2 - (1) + 1]$

$= k^2 + 2k + 1 - k - 1 + 1 - 1$

$= k^2 + k$

$\neq f(k)$

$f(k - 1) + f(1) = (k - 1)^2 - (k - 1) + 1 + [(1)^2 - (1) + 1]$

$= k^2 - 2k + 1 - k + 1 + 1 + 1$

$= k^2 - 3k + 4$

$\neq f(k)$

9. D

By Factor theorem,

$g(-2a) = 0$

$(-2a)^2 + a(-2a) + b = 0$

$4a^2 - 2a^2 + b = 0$

$b = -2a^2$

i.e. $g(x) = x^2 + ax - 2a^2$

$\therefore$ The remainder $= g(2a)$

$= (2a)^2 + a(2a) - 2a^2$

$= 4a^2$

10. A

$y = (h - x)(k - x)$

$= x^2 - (h + k)x + hk$

$a = 1 >$ ➔ The graph opens upwards.

$\therefore$ I is true.

$\Delta = [-(h + k)]^2 - 4hk$

$= h^2 + 2hk + k^2 - 4hk$

$= h^2 - 2hk + k^2$

$= (h - k)^2 > 0$

Note that $hk < 0$ implies $h \neq k$.

$\therefore$ II is true.

The y-intercept $= hk < 0$

$\therefore$ III is not true.

11. C

The interest $= \$88\,000(1 + \frac{6\%}{12})^{4 \times 12} - \$88\,000$

$= \$23\,803$

12. C

Let $x = 8k$ and $y = 5k$.

Then, $2(8k) = 4z - 3(5k)$

$16k = 4z - 15k$

$\therefore$ $z = \frac{31k}{4}$

$y : z = 5k : \frac{31k}{4}$

$= 20 : 31$

13. B

Let $u = \frac{k\sqrt{v}}{w}$ where k is a constant.

$u^2 = \frac{k^2 v}{w^2}$

$\therefore$ I is true.

$u = \frac{k\sqrt{v}}{w}$

$\sqrt{v} = \frac{uw}{k}$

$v = \frac{u^2 w^2}{k^2}$

$\therefore$ II is NOT true.

$w = \frac{k\sqrt{v}}{u}$

$\therefore$ III is true.

14. C

The number of dots in the 7^{th} pattern

$= 8 + [2(1) + 6] + [2(2) + 6] + \ldots + [2(6) + 6]$

$= 8 + 2(1 + 2 + \ldots +6) + 6 \times 6$

$= 8 + \frac{2(1+6)(6)}{2} + 36$

$= 8 + 42 + 36$

$= 86$

15. A

Let r be the common radius. Then, the height of the circular cylinder is $2r$.

The ratio required $= (\frac{1}{2} \times 4\pi r^2 + \pi r^2) : [2\pi r^2 + 2\pi r(2r)]$

$= 3\pi r^2 : 6\pi r^2$

$= 1 : 2$

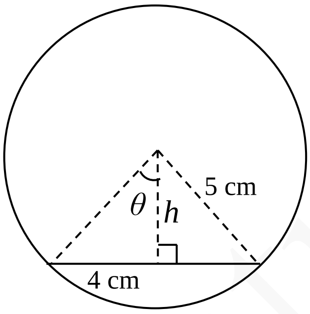
16. D

A perpendicular is drawn from the centre of the circle to the chord as shown. The perpendicular is the bisector of the chord.

By Pythagoras' theorem,

$h = \sqrt{5^2 - 4^2}$

$= 3$ cm



$\sin\theta = \frac{4}{5}$ ➔ $\theta = 53.13^o$

Area of the major segment

$= \pi(5)^2 \times \frac{360^o - 2\times 53.13^o}{360^o} + \frac{3\times 8}{2}$

$= 67$ cm^2 (correct to the nearest cm^2)

17. B

Join PN.

∵ $PM : MQ = 5 : 6$

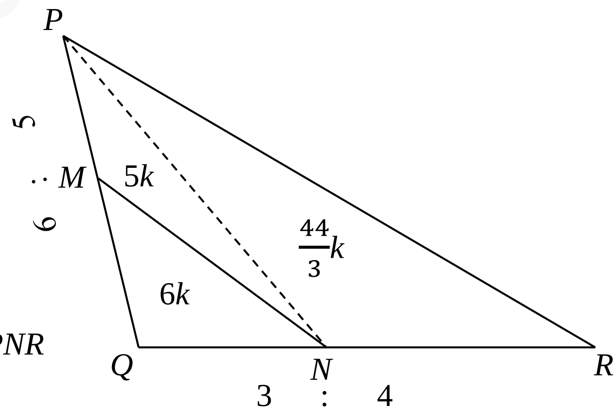
∴ area of $\triangle PMN$: area of $\triangle MQN = 5 : 6$

Let area of $\triangle PMN$ and that of $\triangle MQN$ be $5k$ cm^2 and $6k$ cm^2 respectively. Then,

the area of $\triangle PQN$ = area of $\triangle PMN$ + area of $\triangle MQN$

$= 5k + 6k$

$= 11k$ cm^2



∵ $QN : NR = 3 : 4$

∴ area of $\triangle PQN$: area of $\triangle PNR = 3 : 4$

i.e. $11k$: area of $\triangle PNR = 3 : 4$

➔ Area of $\triangle PNR = \frac{44}{3}k$ cm^2

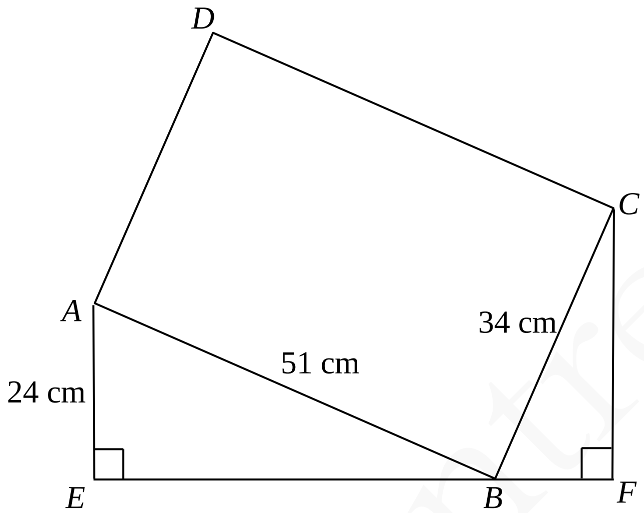
Area of quadrilateral $MNRP$ = area of $\triangle PMN$ + area of $\triangle PNR$

$59 = 5k + \frac{44}{3}k$

➔ $k = 3$

∴ Area of $\triangle MNQ = 6(3) = 18$ cm^2

18. C



$AB = \frac{170}{2} - 34 = 51$ cm

Note that $\triangle AEB \sim \triangle BFC$.

$\frac{BF}{AE} = \frac{BC}{AB}$

$\frac{BF}{24} = \frac{34}{51}$

$BF = 16$ cm

By Pythagoras' Theorem,

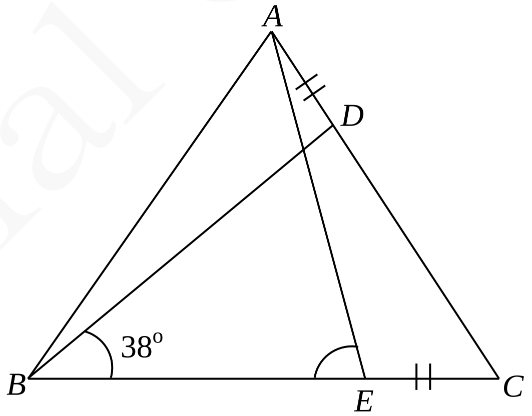
$BE^2 + AE^2 = AB^2$

$BE^2 + 24^2 = 51^2$

$BE = 45$ cm

$\therefore \quad EF = EB + BF = 45 + 16 = 61$ cm

19. D



Note that $\triangle ABD \cong \triangle CAE$.

$\angle ABD = \angle ABE - \angle CBD$

$= 60^\circ - 38^\circ$

$= 22^\circ$

$\angle CAE = \angle ABD$ (corr. $\angle$s, $\cong\triangle$s)

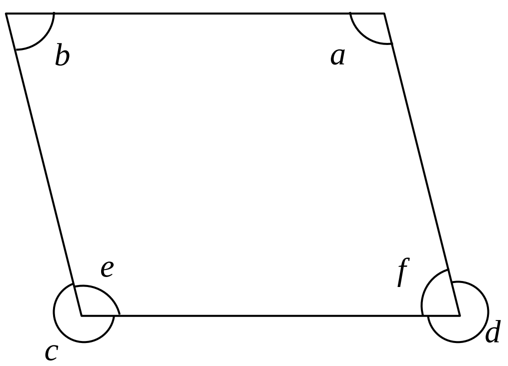
$= 22^\circ$

$\angle AEB = \angle CAE + \angle ACE$

$= 22^\circ + 60^\circ$

$= 82^\circ$

20. B



$a + b = 180^\circ$ (int. $\angle$s, // lines)

$\therefore \quad$ I is true.

$e + c = 360^\circ$ ($\angle$s at a pt.)

$e = 360^\circ - c$

$f + d = 360^\circ$ ($\angle$s at a pt.)

$f = 360^\circ - d$

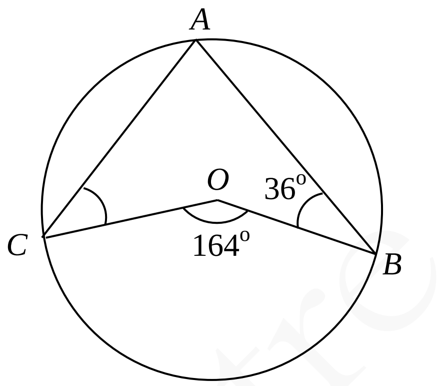
$e + f = 180^\circ$ (int. $\angle$s, // lines)

$(360^\circ - c) + (360^\circ - d) = 180^\circ$

$\Rightarrow \quad c + d = 540^\circ$

$\therefore \quad$ III is true.

21. B



$\angle CAB = \frac{1}{2}\angle COB$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \frac{1}{2}(164^o)$

$= 82^o$

Reflex $\angle COB + \angle COB = 360^o$ ($\angle$s at a pt.)

Reflex $\angle COB + 164^o = 360^o$

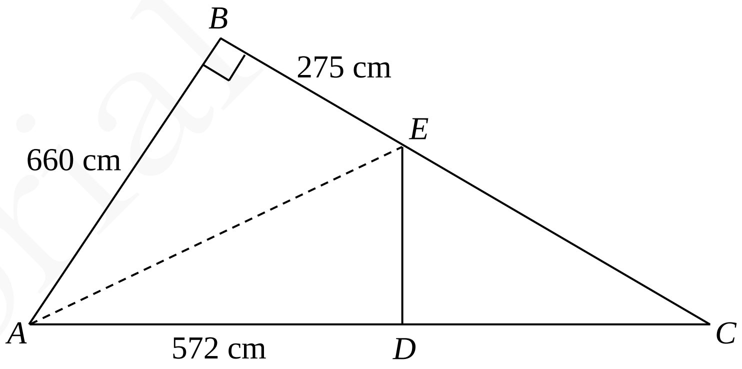
Reflex $\angle COB = 196^o$

$\angle ACO + \angle CAB + \angle ABO +$ reflex $\angle COB = 360^o$ ($\angle$ sum of polygon)

$\angle ACO + 82^o + 36^o + 196^o = 360^o$

$\angle ACO = 46^o$

22. C



$\angle ADE + \angle ABE = 180^o$ (opp. $\angle$s, cyclic quad.)

$\angle ADE + 90^o = 180^o$

$\angle ADE = 90^o$

By Pythagoras' Theorem,

$AE^2 = ED^2 + AD^2 = AB^2 + BE^2$

$\therefore \quad ED^2 + 572^2 = 660^2 + 275^2$

$\Rightarrow \quad ED = 429$ cm

Note that $\triangle ABC \sim \triangle EDC$.

$\frac{CD}{BC} = \frac{EC}{AC} = \frac{ED}{AB}$ (corr. sides, ~$\triangle$s)

$\frac{CD}{275+EC} = \frac{429}{660} = 0.65$

$\Rightarrow \quad CD - 0.65EC = 178.75$ … (1)

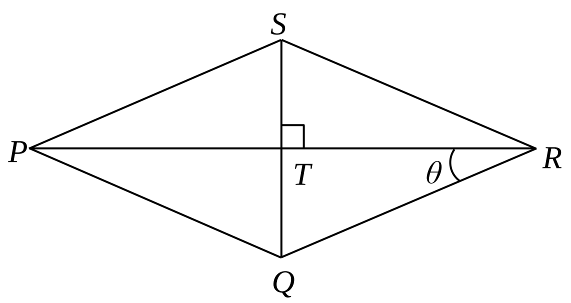
$\frac{EC}{572+CD} = \frac{429}{660} = 0.65$

$\Rightarrow \quad 0.65CD - EC = -371.8$ … (2)

Solving (1) and (2),

$CD = 728$ cm and $EC = 845$ cm

23. C



Note that $PR \perp QS$. (Properties of rhombus)

By symmetry,

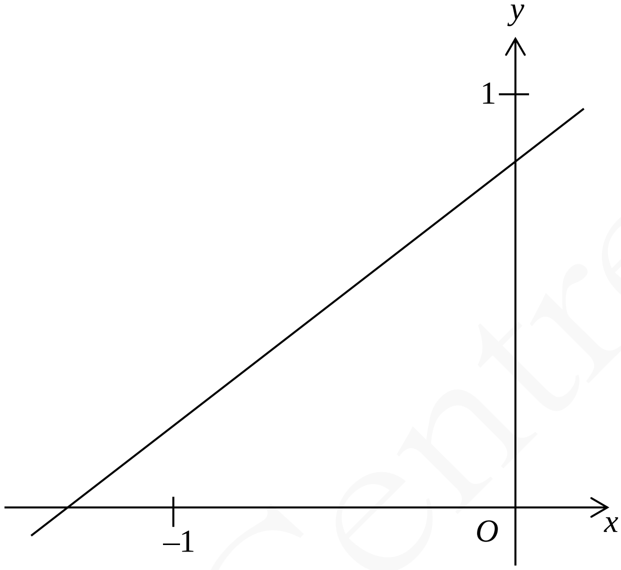
$\angle QPT = \angle QRT = \theta$ and $QT = ST$

$\frac{QT}{PQ} = \sin\theta$

$\frac{PQ}{QT} = \frac{1}{\sin\theta}$ i.e. $\frac{PQ}{ST} = \frac{1}{\sin\theta}$

24. A

Rewrite $mx + ny = 3$ as $y = -\frac{m}{n}x + \frac{3}{n}$



From the graph, $0 <$ the y-intercept $= \frac{3}{n} < 1$

➔ $n > 3$

∴ II is true.

From the graph, slope $= -\frac{m}{n} > 0$

➔ $m < 0$ [∵ $n > 3$]

∴ I is true.

Also, slope $= -\frac{m}{n} < 1$

➔ $-m < n$ [∵ $n > 3$]

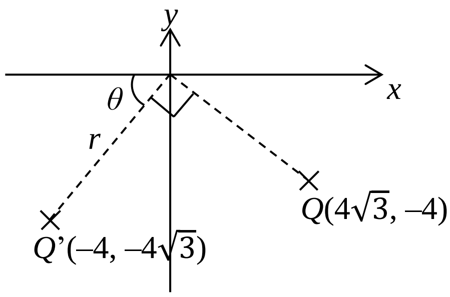
➔ $m + n > 0$

∴ III is true.

25. D

After clockwise rotation, the coordinates of the image of Q(Q') are $(-4, -4\sqrt{3})$.

Refer to the figure on the right.



$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$

➔ $\theta = 60^\circ$

$r = \sqrt{(-4)^2 + (-4\sqrt{3})^2}$

$= 8$

∴ The required polar coordinates are $(8, 180^\circ + 60^\circ)$. i.e. $(8, 240^\circ)$

26. A

When $y = 0$, $x = 5$. i.e. $A = (5, 0)$.

When $x = 0$, $y = -12$ i.e. $B = (0, -12)$.

Let $P = (x, y)$. Then,

$(x - 5)^2 + (y - 0)^2 = (x - 0)^2 + [y - (-12)]^2$

$x^2 - 10x + 25 + y^2 = x^2 + y^2 + 24y + 144$

i.e. $10x + 24y + 119 = 0$

27. C

Let the equation of C be $x^2 + y^2 + Dx + Ey + F = 0$.

Substitute (0, 0) into the equation, we get $F = 0$.

Substitute (10, –24) into the equation,

$10^2 + (-24)^2 + D(10) + E(-24) = 0$

$5D - 12E = -338$ … (1)

Substitute (17, –7) into the equation,

$17^2 + (-7)^2 + D(17) + E(-7) = 0$

$17D - 7E = -338$ … (2)

Solving (1) and (2), we get $D = -10$ and $E = 24$.

∴ The equation of C is $x^2 + y^2 - 10x + 24y = 0$.

Centre of C = $\left(\frac{-(-10)}{2}, \frac{-24}{2}\right)$ = (5, –12)

Radius, $r = \sqrt{5^2 + (-12)^2} = 13$

$PQ = \sqrt{(17-10)^2 + (-7-(24))^2}$

$= \sqrt{338} \neq 26$

∴ PQ is not a diameter.

Alternatively

Slope of OP × slope of OQ

$= \frac{-24}{10} \times \frac{-7}{17}$

$= \frac{84}{85} \neq -1$

∴ PQ is not a diameter.

Area of C

$= \pi(13)^2$

$= 169\pi \neq 196\pi$

The distance between (16, –9) and the centre of C

$= \sqrt{(16-5)^2 + (-9-(-12))^2}$

$= \sqrt{130} < 13$

∴ (16, –9) lies inside C.

Substitute (5, –12) into the left hand side of the equation $5x + 12y = 0$.

$5(5) + 12(-12) = -119 \neq 0$

∴ The centre of C does not lie on the straight line $5x + 12y = 0$.

28. D

Only 532 is divisible by 7.

The required probability

$= \frac{1}{10}$

29. A

Let x be the mean weight of the actresses.

$\frac{63\times 60+40x}{60+40} = 57$

➔ $x = 48$ kg

30. B

∵ The median = 6

∴ There are two possible cases:

1. $x = 6$ (Note : y can be any positive integer.)
2. $x > 6$ and $y \leq 6$

∵ The mean = 6

∴ $2 + 5 + 6 \times 2 + 3x + y = 6 \times 8$

i.e. $3x + y = 29$

For $x = 6$, $y = 11$, then the mode = 6, the range = 11 – 2 = 9 and the variance = 5.25.

For $x = 7$, $y = 8$ which is impossible according to case (2) above.

For $x = 8$, $y = 5$, then the mode = 8, the range = 8 – 2 = 6 and the variance = 3.75.

For $x = 9$, $y = 2$, then the mode = 9, the range = 9 – 2 = 7 and the variance = 7.5

∴ I may not be true.

II is true.

III is not true.

31. C

Consider the common log of the numbers.

$\log (-345)^{768} = 768 \log 345 = 1949$

$\log 453^{-786} = -786 \log 453 = -2088$

$\log \left(\frac{1}{435}\right)^{867} = -867 \log 435 = -2288$

$\log \left(\frac{2}{543}\right)^{876} = 876 \log \frac{2}{543} = -2132$

32. C

Slope of the graph of the linear function = $\frac{6-0}{0-3}$ = –2.

Then, the equation of the linear function is

$\log_a y = -2x + 6$

$y = a^{-2x+6}$

$= a^6(a^{-2})^x$

$\because$ $0 < a < 1$

$\therefore$ $m = a^6 < 1$

$\therefore$ I is true.

$n = a^{-2} > 1$

$\therefore$ II is not true.

$mn^3 = (a^6)(a^{-2})^3$

$= 1$

$\therefore$ III is true.

Alternatively

$\log_a y = \log_a(mn^x)$

$= \log_a m + \log_a n^x$

$= \log_a m + x\log_a n$

$= (\log_a n)x + \log_a m$

Comparing with $y = mx + c$,

$\log_a n = -2$ and $\log_a m = 6$

i.e. $n = a^{-2}$ and $m = a^6$

33. D

$$\begin{cases} \log_4 y = 2x - 1 \ \ldots (1) \\ (\log_4 y)^2 = 20x - 31 \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(2x - 1)^2 = 20x - 31$

$4x^2 - 4x + 1 = 20x - 31$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2$ or 4

$\log_4 y = 2(2) - 1$ or $\log_4 y = 2(4) - 1$

$\log_4 y = 3$ or 7

$\frac{\log_2 y}{\log_2 4}$ = 3 or 7 [using change of base formula]

$\log_2 y = 6$ or 14

Alternatively

From (1), $2x = \log_4 y + 1 \ \ldots (3)$

Substitute (3) into (2),

$(\log_4 y)^2 = 10(\log_4 y + 1) - 31$

$(\log_4 y)^2 - 10\log_4 y + 21 = 0$

$(\log_4 y - 3)(\log_4 y - 7) = 0$

$\log_4 y = 3$ or 7

34. A

$12B00CD000000E_{16}$

$= 1 \times 16^{13} + 2 \times 16^{12} + 11 \times 16^{11} + 12 \times 16^8 + 13 \times 16^7 + 14$

$= 1 \times (4^2)^{13} + 2 \times (4^2)^{12} + 11 \times (4^2)^{11} + 12 \times (4^2)^8 + 13 \times (4^2)^7 + 14$

$= 4^{26} + 2 \times 4^{24} + 11 \times 4^{22} + 12 \times 4^{16} + 13 \times 4^{14} + 14$

$= (4^4 + 2 \times 4^2 + 11) \times 4^{22} + (12 \times 4^2 + 13) \times 4^{14} + 14$

$= 299 \times 4^{22} + 205 \times 4^{14} + 14$

35. B

$z = 4 + 5i^{10} - ki^{15} + 6i^{21} + 2ki^{28}$

$= 4 + 5i^{4 \times 2 + 2} - ki^{4 \times 3 + 3} + 6i^{4 \times 5 + 1} + 2ki^{4 \times 7}$

$= 4 + 5i^2 - ki^3 + 6i + 2k$

$= 4 - 5 + 2k - k(-i) + 6i$

$= 2k - 1 + (k + 6)i$

$\therefore \quad 2k - 1 = k + 6$

$k = 7$

$\therefore$ The real part of $z = 13$

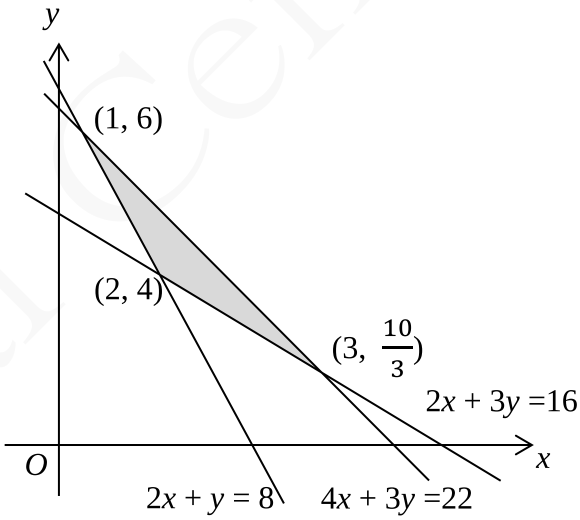
36. B

$$\begin{cases} 2x + y = 8 \ \ldots \ (1) \\ 2x + 3y = 16 \ \ldots \ (2) \\ 4x + 3y = 22 \ \ldots \ (3) \end{cases}$$

Solving (1) and (2), the intersection of (1) and (2) is (2, 4).

Similarly, the intersection of (1) and (3) is (1, 6)

while that of (2) and (3) is $(3, \frac{10}{3})$.



Let $P(x, y) = 7x + 6y$

$P(2, 4) = 7(2) + 6(4) = 38$

$P(1, 6) = 7(1) + 6(6) = 43$

$P(3, \frac{10}{3}) = 7(3) + 6(\frac{10}{3}) = 41$

$\therefore$ The least value of $7x + 6y$ is 38.

37. D

$\frac{1}{8p^2} = 27p$

$p^3 = \frac{1}{216}$

$p = \frac{1}{6}$

$\therefore$ common ratio, $r = 27p = \frac{9}{2}$

$a_4 = a_3 \times r$

$= 27 \times \frac{1}{6} \times \frac{9}{2}$

$= \frac{81}{4}$

38. D

Join AB and BD.

$\angle ABD = \angle DAP = 42^\circ$ ($\angle$ in alt. segment)

$\angle BDC = \angle CBQ = 39^\circ$ ($\angle$ in alt. segment)

$\angle ADB = \angle ADC - \angle BDC$

$= 79^\circ - 39^\circ$

$= 40^\circ$

$\angle BAD + \angle ABD + \angle ADB = 180^\circ$ ($\angle$ sum of $\triangle$)

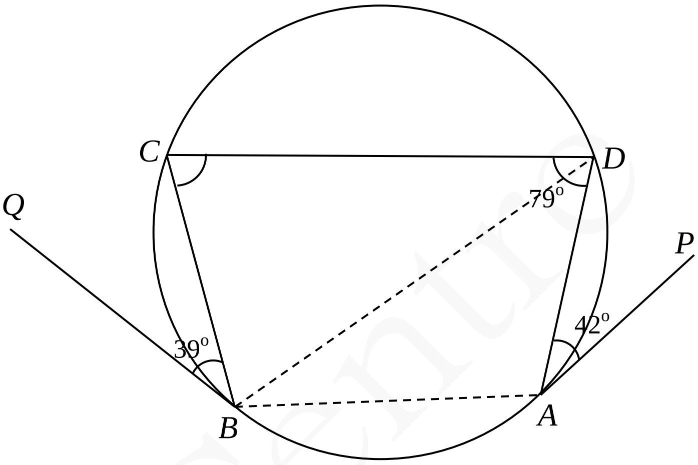
$\angle BAD + 42^\circ + 40^\circ = 180^\circ$

$\angle BAD = 98^\circ$

$\angle BCD + \angle BAD = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle BCD + 98^\circ = 180^\circ$

$\angle BCD = 82^\circ$



39. C

$\sin^2 x = 6\cos^2 x$

$\tan^2 x = 6$

$\tan x = \sqrt{6}$ or $-\sqrt{6}$

$x = 67.8^\circ, 112^\circ, 248^\circ$ or 292°

40. A

Note that $\alpha = \angle GFH = 45^\circ$ and $\beta = 90^\circ$.

$\therefore \quad \alpha < 60^\circ < \beta$

41. A

$\because \quad \angle AOB = 90^\circ$

$\therefore$ AB is a diameter of the circle passing through A, O and B. (Converse of $\angle$ in a semi-circle)

i.e. The midpoint of AB is the circumcentre of ΔOAB.

$\therefore$ Circumcentre $= (\frac{a}{2}, \frac{b}{2})$

Then, $4(\frac{a}{2}) + 16(\frac{b}{2}) = 17a$

$2a + 8b = 17a$

$15a = 8b$

$\frac{a}{b} = \frac{8}{15}$

i.e. $a : b = 8 : 15$

42. B

Number of passwords $= 5! \times 2!$

$= 240$

43. B

The girl draws a white ball only if in the previous turns, they draw only red balls. Otherwise, the draw is ended. Furthermore, the girl can only draw a white ball in the 2nd draw, the 4th draw etc.

The required probability

$$= \frac{3}{7}\times\frac{2}{7} + \frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}\times\frac{2}{7} + \frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}\times\frac{2}{7} + \ldots$$

$$= \frac{3}{7}\times\frac{2}{7}\times\left[1+\left(\frac{3}{7}\right)^2+\left(\frac{3}{7}\right)^4+\cdots\right]$$

$$= \frac{6}{49}\times\frac{1}{1-\left(\frac{3}{7}\right)^2}$$

$$= \frac{3}{20}$$

44. C

$x = 30(1 + 50\%) + 8 = 53$

Let μ and σ be the mean and standard deviation of the test scores before adjustment respectively. Let x' be the test score of the student. Then,

$$\frac{x'-\mu}{\sigma} = -2$$

$\sigma = 4$

Now, the test score of the student after adjustment $= x'(1 + 50\%) + 8 = 1.5x' + 8$

The mean of the test scores after adjustment $= \mu(1 + 50\%) + 8 = 1.5\mu + 8$

The standard deviation of the test scores after adjustment $= \sigma(1 + 50\%) = 1.5\sigma$

$$\therefore \quad z = \frac{1.5x' + 8-(1.5\mu + 8)}{1.5\sigma}$$

$$= \frac{x'-\mu}{\sigma}$$

$$= -2$$

45. D

The mean of $S_1 = \dfrac{d-6+d-2+d-1+d+3+d+5+d+7}{6} = d + 1$

The mean of $S_2 = \dfrac{d-7+d-5+d-3+d+1+d+2+d+6}{6} = d - 1$

$\therefore$ I is not true.

Note that the numbers in S_1 and S_2 are separated by the same amount. So, their standard deviations are equal.

$\therefore$ II is true.

The inter-quartile range of $S_1 = d + 5 - (d - 2) = 7$

The inter-quartile range of $S_2 = d + 2 - (d - 5) = 7$

$\therefore$ III is true.