Suggested Solution for 2024 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$(x + 3y)^2 - (x - 3y)^2$

$= [(x + 3y) + (x - 3y)][(x + 3y) - (x - 3y)]$

$= (2x)(6y)$

$= 12xy$

2. D

$\frac{(2\alpha)^3}{(4\alpha^{-5})^{-1}}$

$= \frac{2^3\alpha^3}{4^{-1}\alpha^5}$

$= \frac{8\cdot 4}{\alpha^{5-3}}$

$= \frac{32}{\alpha^2}$

3. A

$k = \frac{5}{2m} + n$

$\frac{5}{2m} = k - n$

$\frac{2m}{5} = \frac{1}{k-n}$

$m = \frac{5}{2(k-n)}$

Alternatively

$k = \frac{5}{2m} + n$

$2km = 5 + 2mn$

$2km - 2mn = 5$

$2m(k - n) = 5$

$m = \frac{5}{2(k-n)}$

4. A

$\sqrt{333} \approx 18.24828759\ldots$

$\approx$ 18 (correct to the nearest integer)

$\approx$ 18.25 (correct to 2 decimal places)

$\approx$ 18.2 (correct to 3 significant figures)

$\approx$ 18.2483 (correct to 4 decimal places)

5. B

Let $\$x$ and $\$y$ be the price of an apple and a lemon respectively.

$2x + 3y = 38 \ldots (1)$

$3x + 2y = 47 \ldots (2)$

Solving (1) and (2),

$x = 13$ and $y = 4$

$\therefore$ The price of 4 apples and 7 lemons

$= \$(4x + 7y)$

$= \$[4(13) + 7(4)]$

$= \$80$

6. A

$4x^2 + 2ax + 3a \equiv x(4x + b) + 2c$

$\equiv 4x^2 + bx + 2c$

$\therefore$ $2a = b$ and $3a = 2c$

i.e. $a : b = 1 : 2 = 2 : 4$ and $a : c = 2 : 3$

$\therefore$ $a : b : c = 2 : 4 : 3$

7. B

$x^2 - 3x = (m-1)^2 - 3(m-1)$

$x^2 - (m-1)^2 - 3x + 3(m-1) = 0$

$[x + (m-1)][x - (m-1)] - 3[x - (m-1)] = 0$

$[x - (m-1)][x + (m-1) - 3] = 0$

$[x - (m-1)][x + (m-4)] = 0$

$x = m - 1$ or $x = 4 - m$

<u>Alternatively</u>

$x^2 - 3x = (m-1)^2 - 3(m-1)$

$x^2 - 3x = (m-1)(m-1-3) = (m-1)(m-4)$

$x^2 - 3x - (m-1)(m-4) = 0$

By cross method,

$x \quad -(m-1)$

$x \quad (m-4)$

$[x - (m-1)][x + (m-4)] = 0$

$x = m - 1$ or $x = 4 - m$

By quadratic formula,

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4[-(m-1)(m-4)]}}{2}$$

$$= \frac{3 \pm \sqrt{4m^2 - 20m + 25}}{2}$$

$$= \frac{3 \pm \sqrt{(2m-5)^2}}{2}$$

$$= \frac{3 + (2m-5)}{2} \text{ or } \frac{3 - (2m-5)}{2}$$

$= m - 1$ or $x = 4 - m$

8. D

g(1) = g(2)

➔ $(1 + 1)(1 + a) = (2 + 1)(2 + a)$

➔ $a = -4$

$\therefore$ $g(-4) = (-4 + 1)(-4 + -4) = 24$

9. C

By Factor theorem,

f(–k) = 0

i.e. $(-k)^3 + k(-k)^2 + 5(-k) + 10 = 0$

➔ $k = 2$

∴ $f(x) = x^3 + 2x^2 + 5x + 10$

By Remainder theorem,

Remainder = f(–1)

$= (-1)^3 + 2(-1)^2 + 5(-1) + 10$

$= 6$

10. B

$\frac{1-x}{2} \geq 4$ or $7 + 5x \leq -3$

$1 - x \geq 8$ or $5x \leq -10$

$x \leq -7$ or $x \leq -2$

∴ $x \leq -2$

11. C

Let x be the number of students in the school.

$x \times 40\% \times \beta\% + x(1 - 40\%) \times 30\% = x \times 40\%$

$\beta = 55$

12. A

Average speed $= \frac{60\times18+40\times27}{18+27}$

$= 48$ km/h

13. C

Let $z = \frac{kx^2}{y}$ where k is a constant. Let z' be the new value of z. Then,

$z' = \frac{k[(1+20\%)x]^2}{(1-20\%)y}$

$= \frac{1.8kx^2}{y}$

$= 1.8z$

% change in z

$= \frac{1.8z-z}{z} \times 100\%$

$= 80\%$

14. A

$y = 2(6 - x)^2 - 7$

$\quad = 2(x - 6)^2 - 7$

$\because \quad a > 0$

$\therefore$ The graph opens u[wards.

Note that:

Vertex = (6, –7) ➔ The graph cuts the x-axis at two points. i.e. B is not true.

Substitute $x = 0$, y-intercept $= 2(0 - 6)^2 - 7 = 65 \neq -7$ i.e. C is not true.

$2[6 - (-6)]^2 - 7 = 281 \neq -7$ i.e. D is not true.

15. D

Let r cm and θ cm be the radius and the angle of the sector respectively.

$\pi r^2 \times \dfrac{\theta}{360^o} = 80\pi \ldots (1)$

$2\pi r \times \dfrac{\theta}{360^o} = 8\pi \ldots (2)$

Solving (1) and (2), $r = 20$ and $\theta = 72^o$

16. D

Let $32k$ and $15k$ be the heights of the right circular cylinder and the right circular cone respectively.

Let r cm be the base radius of the circular cone.

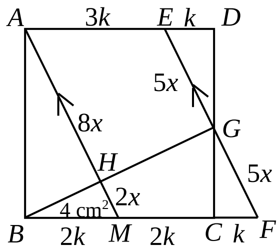
$$\frac{\frac{1}{3}\pi r^2(15k)}{\pi(25)^2(32k)} = \frac{9}{10}$$

$r = 60$

17. C

Note that $AMFE$ is a parallelogram. Let $ED = k$. Then, $AE = 3k$, $BM = MC = 2k$ and $CF = k$.

Note also that $\triangle BHM \sim \triangle BGF$.



$$\frac{HM}{GF} = \frac{BM}{BF} = \frac{2k}{2k+2k+k} = \frac{2}{5}$$

$$\frac{Area\ of\ \Delta BGF}{Area\ of\ \Delta BHM} = \left(\frac{GF}{HM}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$\therefore$ Area of $\triangle BGF = 25$ cm^2

Then, area of trapezium $FGHM$ = area of $\triangle BGF$ – area of $\triangle BHM = 25 - 4 = 21$ cm^2

Note that $\triangle DEG \cong \triangle CFG$ and G is the mid-point of EF.

Let $EG = GF = 5x$. Then, $HM = 2x$ and $AH = 8x$.

Let h cm be the common height of trapezium $AEGH$ and $FGHM$. Then,

$$\frac{Area\ of\ trapezium\ AEGH}{Area\ of\ trapezium\ FGHM} = \frac{(5k+8k)h/2}{(2k+5k)h/2} = \frac{13}{7}$$

$\therefore$ Area of trapezium $AEGH = 21 \times \dfrac{13}{7} = 39$ cm^2

18. C

$\because$ $BC^2 + BD^2 = 5^2 + 12^2 = 13^2 = CD^2$

$\therefore$ $\angle CBD$ is a right $\angle$.(Converse of Pythagoras' Theorem)

By Pythagoras' Theorem,

$AB^2 + BD^2 = AD^2$

$AB^2 + 12^2 = 37^2$

$AB = 35$

i.e. $AC = 5 + 35 = 40$ cm

By Pythagoras' Theorem,

$AC^2 + CE^2 = AE^2$

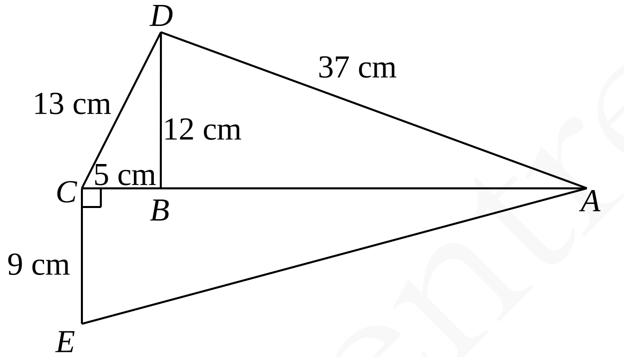
$40^2 + 9^2 = AE^2$

$AE = 41$

$\therefore$ Perimeter of $ADCE = AD + DC + CE + EA$

$= 37 + 13 + 9 + 41$

$= 100$ cm



19. D

Draw parallel lines as shown in the left.

According to the diagram on the left,

$a + p = 360^o$ ($\angle$s at a pt.)

$a = 360^o - p$

$b = a = 360^o - p$ (alt. $\angle$s, // lines)

$c = q - b = q - (360^o - p) = p + q - 360^o$

$d = c = p + q - 360^o$ (alt. $\angle$s, // lines)

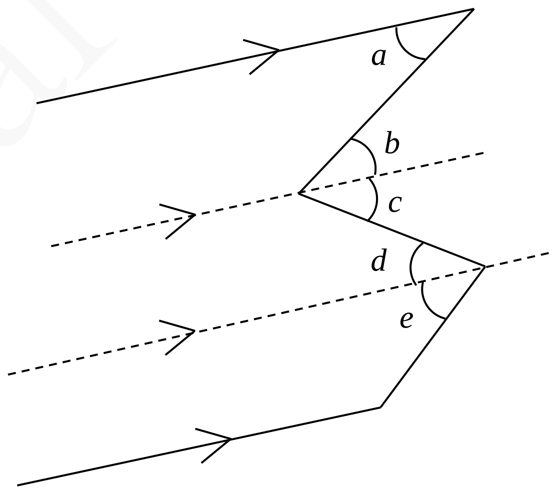
$e + s = 180^o$ (int. $\angle$s, // lines)

$e = 180^o - s$

$d + e + r = 360^o$ ($\angle$s at a pt.)

$(p + q - 360^o) + (180^o - s) + r = 360^o$

i.e. $p + q + r - s = 540^o$



20. D

Let n be the number of sides of the polygon.

$180^o \times (n - 2) = 900^o$

$n = 7$

The number of diagonals $= \frac{7\times6}{2} = 21$

$\therefore$ I is NOT true.

II and III are true for a regular heptagon.

21. B

Let $\angle FCH = x$ and $\angle GCH = y$. Note that $x + y = 90^{\circ}$.

Then, $\angle IFC = \angle FCH = x$ (alt. $\angle$s, BH // EF)

$\angle BCE = \angle GCH = y$ (vert. opp. $\angle$s)

$\angle DCE = \angle BCE = y$ (properties of rhombus)

$\angle FCD + \angle DCE = 90^{\circ}$

➔ $\angle FCD = 90^{\circ} - y = x = \angle IFC$

$\therefore$ $CI = FI$ (sides opp. equal $\angle$s)

$\therefore$ I is true.

$AC \perp DB$ (properties of rhombus)

i.e. $\angle CEB = 90^{\circ}$

$\angle CBE + \angle CEB + \angle BCE = 180^{\circ}$ ($\angle$ sum of Δ)

$\angle CBE + 90^{\circ} + y = 180^{\circ}$

➔ $\angle CBE = 90^{\circ} - y = x$

$\angle ABE = \angle CBE = x$ (properties of rhombus)

but $\angle GCH = y$

$\therefore$ II may not be true.

Note that owing to the properties of rhombus, ΔADE, ΔABE, ΔCDE and ΔCBE are congruent.

Note that ΔCBE and ΔEFC are congruent(ASA).

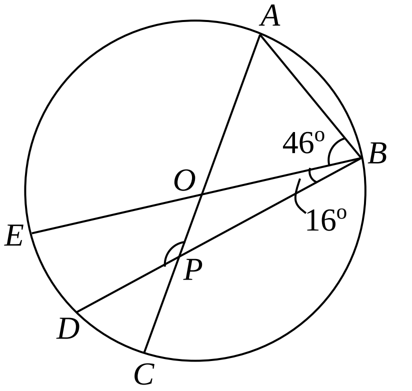
Note that ΔEFC and ΔHCF are congruent(ASA).

$\therefore$ $\Delta ADE \cong \Delta HCF$

$\therefore$ III is true.

22. B

Mark the intersection of AC and BE as O which is the centre of the circle.



$\angle OAB = \angle OBA = 46^{\circ}$ (base $\angle$s, isos. Δ)

$\angle POB = \angle OBA + \angle OAB$ (ext. $\angle$ of Δ)

$= 46^{\circ} + 46^{\circ}$

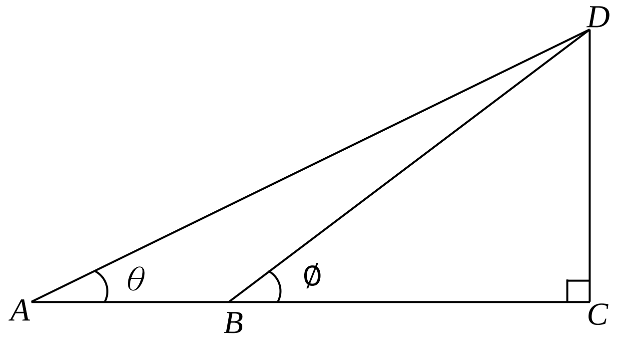
$= 92^{\circ}$

$\angle APD = \angle POB + \angle DBE$ (ext. $\angle$ of Δ)

$= 92^{\circ} + 16^{\circ}$

$= 108^{\circ}$

23. A



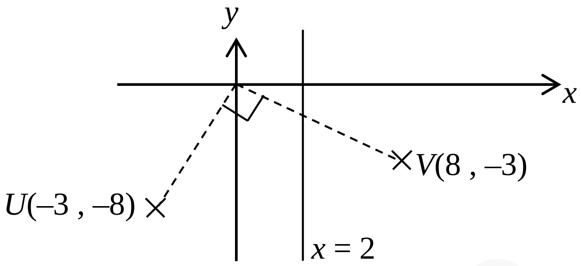
$\frac{DC}{AD} = \sin\theta$ … (1)

$\frac{DC}{BC} = \tan\emptyset$ … (2)

Combining (1) and (2),

$\frac{BC}{AD} = \frac{\sin\theta}{\tan\emptyset}$

24. A



After clockwise rotation, the coordinates of V are (8, –3).

Let the x-coordinate of W be x'.

$2 - x' = 8 - 2$

➔ $x' = -4$

25. C

Let $P = (x, y)$. Then,

$x - y + 13 = 0$ … (1) [$\because$ P lies on the straight line]

$\because$ $AP = PB$

$\therefore$ $[x - (-3)]^2 + (y - 1)^2 = [x - (-7)]^2 + [y - (-5)]^2$

➔ $2x + 3y + 16 = 0$ … (2)

Solving (1) and (2), we have $x = -11$ and $y = 2$

26. B

Rewrite the equations of the straight lines in slope-intercept form.

$$\begin{cases} y = \frac{3}{4}x - \frac{7k}{8} \\ y = -\frac{k}{12}x + \frac{5}{12} \end{cases}$$

$\therefore$ If the two lines are parallel and the y-intercepts of them are not equal, then they do not intersect with each other.

i.e. $-\frac{k}{12} = \frac{3}{4}$

➔ $k = -9$

Check that the y-intercepts of the straight lines are $-\frac{7(-9)}{8}$ i.e. $\frac{63}{8}$ and $\frac{5}{12}$ which are not equal.

27. D

Note that $C : x^2 + y^2 - 2x + 4y - \frac{4}{3} = 0$. Let G be the centre of the circle. Then,

$G = \left(\frac{-(-2)}{2}, \frac{-4}{2}\right) = (1, -2)$

Radius, $r = \sqrt{(1)^2 + (-2)^2 - \left(-\frac{4}{3}\right)} = \sqrt{\frac{19}{3}}$

$OG = \sqrt{(1-0)^2 + (-2-0)^2} = \sqrt{5} < \sqrt{\frac{19}{3}}$

$\therefore$ O lies inside C.

$\therefore$ I is true.

The circumference of $C = 2\pi\sqrt{\frac{19}{3}} < 16$

$\therefore$ II is true.

$\because$ The y-coordinate of $G = -2$

$\therefore$ III is true.

28. C

Refer to the table on the right. The numbers circled are not less than 12.

The required probability

$= \frac{7}{15}$

1st card

2nd card

	1	2	3	4	5	6
1	X	2	3	4	5	6
2	X	X	6	8	10	12
3	X	X	X	12	15	18
4	X	X	X	X	20	24
5	X	X	X	X	X	30

29. B

From the box-and-whisker diagram, range = 472 – 136 = 336

Inter-quartile range = m – 163

$\therefore$ 3(m – 163) 336

➔ $m = 275$

30. D

$$\frac{5+5+5+6+9+9+11+13+m+n}{10} = 7$$

$\therefore \quad m + n = 7$

Since m and n cannot be 6, 9, 11 or 13 at the same time, the mode must be 5.

$\therefore \quad$ II is true.

Possible values of m and n and the corresponding values of the median and standard deviation are shown below:

m	n	median	standard deviation
1	6	6	3.31662479
2	5	5.5	3.193743885
3	4	5.5	3.130495168

$\therefore \quad$ Both II and III are true.

31. B

Taking the smallest degree from u, v and w, the H.C.F. is u^2vw.

32. A

$AF000000000BC_{16}$

$= 10 \times 16^{12} + 15 \times 16^{11} + 11 \times 16^{1} + 12 \times 16^{0}$

$= (10 \times 16 + 15) \times 16^{11} + 188$

$= 175 \times 16^{11} + 188$

33. B

$$\begin{cases} x = \log_2 y - 2 \ \ldots \ (1) \\ (\log_2 y)^2 = 5\log_2 y + x - 7 \ \ldots \ (2) \end{cases}$$

Substitute (1) into (2),

$(\log_2 y)^2 = 5\log_2 y + (\log_2 y - 2) - 7$

$(\log_2 y)^2 - 6\log_2 y + 9 = 0$

$(\log_2 y - 3)^2 = 0$

$\log_2 y = 3$

$y = 8$

34. D

Slope of the graph = –16

Using slope-intercept form of a straight line equation(i.e. $y = mx + c$), we get

$y^3 = -16\sqrt{x} + 32$

When $x = 36$,

$y^3 = -16\sqrt{36} + 32$

$\quad = -64$

$y = -4$

35. A

$z = (a-5)i + \frac{(a+2)i}{2+i}$

$= (a-5)i + \frac{(a+2)i}{2+i} \times \frac{2-i}{2-i}$

$= (a-5)i + \frac{2ai+4i-ai^2-2i^2}{2^2+1}$

$= (a-5)i + \frac{a+2+(2a+4)i}{5}$

$= \frac{a+2}{5} + \frac{5(a-5)+2a+4}{5}i$

$= \frac{a+2}{5} + \frac{7a-21}{5}i$

$\because$ z is a real number.

$\therefore$ $\frac{7a-21}{5} = 0$

$a = 3$

Then, $z = \frac{3+2}{5} = 1$

$a - z = 2$

36. C

Let T(n) be the nth term of the sequence. Then,

$T(n) = S(n) - S(n-1)$

$= n(2n+3) - (n-1)[2(n-1)+3]$

$= 4n + 1$

$\therefore$ II is true.

Note that $T(n) - T(n-1) = 4n + 1 - [4(n-1)+1] = 4$

$\therefore$ The sequence is an arithmetic sequence.

$\therefore$ III is true.

$4n + 1 = 14$

$n = \frac{13}{4}$ is not an integer.

$\therefore$ I is NOT true.

37. C

$$\begin{cases} x - 2y = 1 \ \dots (1) \\ x + 4y = 13 \ \dots (2) \\ 2x - y = -1 \ \dots (3) \end{cases}$$

Solving (1) and (2), the intersection of (1) and (2) is (5, 2).

Similarly, the intersection of (1) and (3) is (–1, –1)

while that of (2) and (3) is (1, 3).

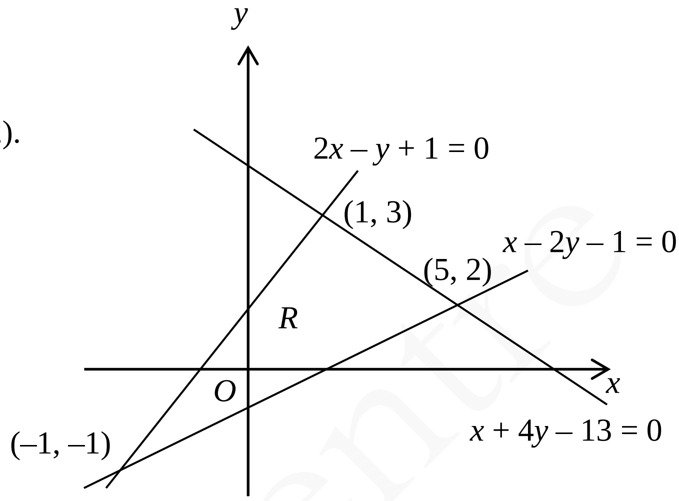
Let $P(x, y) = 5x - 2y + c$

$P(5, 2) = 5(5) - 2(2) + c \geq 22 \Rightarrow c \geq 1$

$P(-1, -1) = 5(-1) - 2(-1) + c \geq 22 \Rightarrow c \geq 25$

$P(1, 3) = 5(1) - 2(3) + c \geq 22 \Rightarrow c \geq 23$

$\therefore$ Combining the results, $c \geq 25$.



38. B

Join CD.

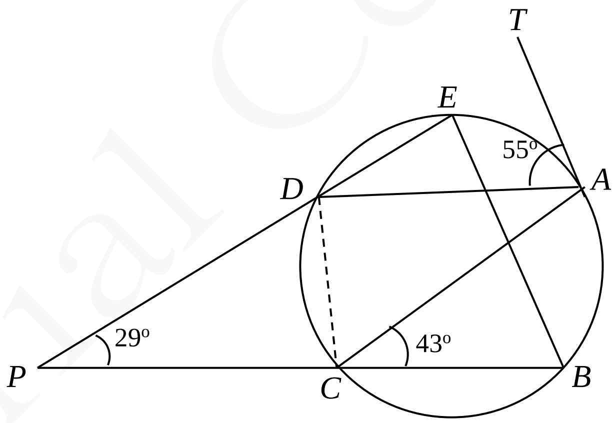
$\angle ACD = \angle DAT = 55^o$ ($\angle$ in alt. segment)

$\angle CDP + \angle CPD = \angle DCB$ (ext. $\angle$ of Δ)

$\angle CDP + 29^o = 55^o + 43^o$

$\angle CDP = 69^o$

$\angle CBE = \angle CDP = 69^o$ (ext. $\angle$ = int. opp. $\angle$)



39. A

$4\cos^2\theta - 3\cos\theta - 1 = 0$

$(4\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{4}$ or 1

$\theta = 105^o, 256^o$ or 360^o for $0^o < \theta \leq 360^o$.

40. C

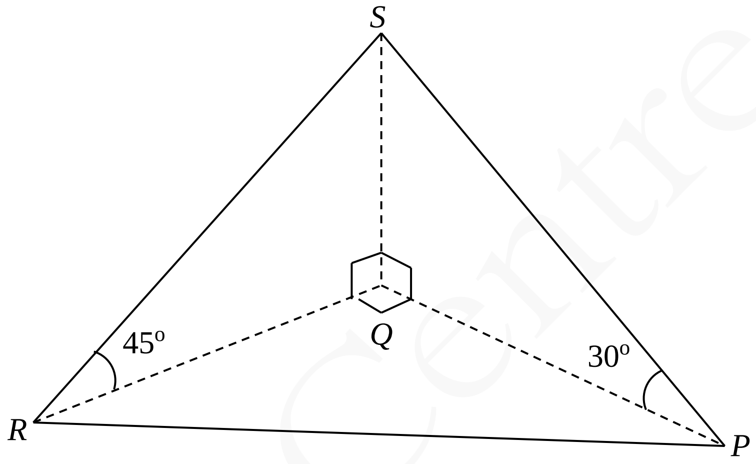
Let the length of SQ be h. Then,

$\frac{SQ}{RQ} = \tan\angle QRS$

$\frac{h}{RQ} = \tan 45^\circ$ i.e. $RQ = h$

$\frac{SQ}{PQ} = \tan\angle QPS$

$\frac{h}{PQ} = \tan 30^\circ$ i.e. $PQ = \sqrt{3}h$



By Pythagoras's Theorem,

$PR^2 = PQ^2 + RQ^2$

$= (\sqrt{3}h)^2 + (h)^2$

$= 4h^2$

$\frac{SQ}{PS} = \sin\angle QPS$

$\frac{h}{PS} = \sin 30^\circ$ i.e. $PS = 2h$

$\frac{SQ}{RS} = \sin\angle QRS$

$\frac{h}{RS} = \sin 45^\circ$ i.e. $RS = \sqrt{2}h$

By cosine formula,

$PS^2 = PR^2 + RS^2 - 2(PR)(RS)\cos\angle PRS$

$(2h)^2 = 4h^2 + \left(\sqrt{2}h\right)^2 - 2(2h)\left(\sqrt{2}h\right)\cos\angle PRS$

$\cos\angle PRS = \frac{\sqrt{2}}{4}$

41. A

$\angle QPR = 136^\circ$

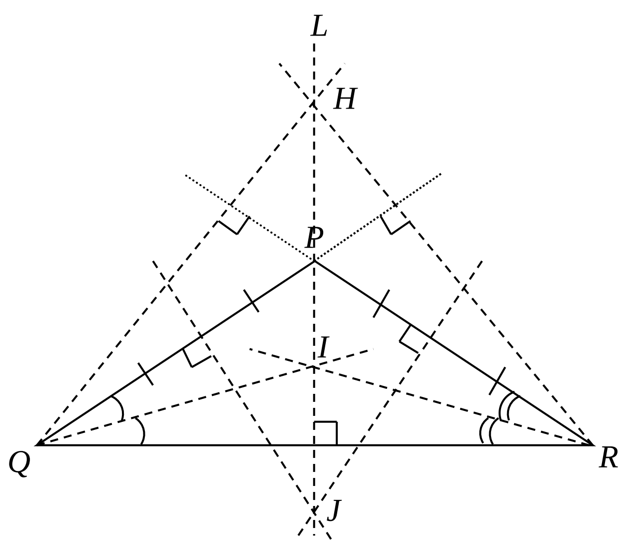
Refer to the diagram on the right.

Note that G, H, I and J lies on the same straight line L.

I is true.

II is true.

III is NOT true.



42. C

Referring to the diagram on the right, the managers can take any two positions indicated by the arrows.

Number of differeent queues

$= P_2^8 \times P_7^7$

= 282 240

<u>Alternatively</u>

Suppose the two managers are next to each other. Taking them as one person. It will be a queue of 8 persons. In addition, the 2 managers can interchange their positions. Therefore, the number of queues the two mangers are next to each other = $P_8^8 \times P_2^2$

Hence, the number of queues the managers are not next to each other

$= P_9^9 - P_8^8 \times P_2^2$

= 282 240

43. D

The required probability

= 1 – P("all wrong")

= 1 – (1 – 0.6)(1 – 0.7)(1 – 0.8)

= 0.976

44. D

Rearrange the scores in ascending order.

10 13 16 17 19 25 26 28 30 30 32 39

Median = $\frac{25+26}{2}$ = 25.5

∴ I is NOT true.

Mean, $\bar{x} = \frac{10+13+16+17+19+25+26+28+30+30+32+39}{12}$ = 23.75

Standard deviation, $\sigma \approx$ 8.347903929

∴ III is true.

The standard score of the largest datum(39) $\approx \frac{39-23.75}{8.347903929} \approx$ 1.826805882 < 2

∴ II is true.

45. B

New variance = $9^2 \times 16$

New standard deviation = $\sqrt{9^2 \times 16}$

= 36