Suggested Solution for 2025 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$\frac{(27x)^5}{(3x^{-2})^4}$

$= \frac{(3^3x)^5}{3^4x^{-8}}$

$= \frac{3^{15}x^5x^8}{3^4}$

$= 3^{15-4}x^{5+8}$

$= 3^{11}x^{13}$

2. B

$36 - (3m + 4n)^2$

$= [6 + (3m + 4n)][6 - (3m + 4n)]$

$= (6 + 3m + 4n)(6 - 3m - 4n)$

3. D

$(x + 8)(x + a) + b \equiv x^2 + 5a(x + 3)$

$x^2 + (8 + a)x + 8a + b \equiv x^2 + 5ax + 15a$

By comparing the coefficient of x,

$8 + a = 5a$

$a = 2$

By comparing the constant term,

$8(2) + b = 15(2)$

$b = 14$

Alternatively

$(x + 8)(x + a) + b \equiv x^2 + 5a(x + 3)$

Put $x = -8$,

$b = 64 - 25a$ i.e. $25a + b = 64$ … (1)

Put $x = -3$,

$5(-3 + a) + b = 9$ i.e. $5a + b = 24$ … (2)

Solving (1) and (2),

we have $a = 2$ and $b = 14$

4. A

$(3c + 1)(d - 4) = 2d(5c - 1)$

$3cd + d - 12c - 4 = 10cd - 2d$

$7cd + 12c = 3d - 4$

$c(7d + 12) = 3d - 4$

$c = \frac{3d-4}{7d+12}$

5. A

$x^2 + 4x = k^2 - 2k - 3$

$x^2 + 4x - (k - 3)(k + 1) = 0$

By cross method,

$$\begin{array}{ccc} x & & k+1 \\ & \times & \\ x & & -(k-3) \end{array}$$

$[x - (k - 3)][x + (k + 1)] = 0$

$x = k - 3$ or $x = -k - 1$

<u>Alternatively</u>

By quadratic formula,

$$x = \frac{-4 \pm \sqrt{4^2 - 4[-(k-3)(k+1)]}}{2}$$

$$= \frac{-4 \pm \sqrt{4k^2 - 8k + 4}}{2}$$

$$= \frac{-4 \pm \sqrt{4(k-1)^2}}{2}$$

$$= \frac{-4 + (2k-2)}{2} \text{ or } \frac{-4 - (2k-2)}{2}$$

$= k - 3$ or $x = -k - 1$

6. D

Note that for correcting to 2 decimal places, the absolute error = $0.01 \div 2 = 0.005$

The range will be 5.67 ± 0.005 i.e. $5.665 \leq x < 5.675$.

7. D

$4y + 1 < 5y - 3 \leq 8y - 9$

$4y + 1 < 5y - 3$ and $5y - 3 \leq 8y - 9$

$y > 4$ and $3y \geq 6$

$y > 4$ and $y \geq 2$

$\therefore \quad y > 4$

8. C

$f(4) + f(-4) = 38$

$\Rightarrow \quad 4^2 + 7(4) + k + (-4)^2 + 7(-4) + k = 38$

$\Rightarrow \quad k = 3$

9. D

By Factor theorem,

$p(-3) = 0$

i.e. $\quad n(-3)^3 - 3n(-3) + 36 = 0$

$\Rightarrow \quad n = 2$

$\therefore \quad p(3) = 2(3)^3 - 3(2)(3) + 36$

$\qquad = 72$

10. C

The amount

$= \$40\,000(1 + \frac{3\%}{2})^{5\times 2}$

$= \$46\,422$ (correct to the nearest dollar)

11. A

$\frac{\alpha+2\beta}{\gamma+2\alpha} = \frac{4}{5}$

➔ $4\gamma = 10\beta - 3\alpha$ … (1)

$\frac{\alpha+2\beta}{\beta+2\gamma} = \frac{4}{9}$

➔ $8\gamma = 9\alpha + 14\beta$ … (2)

Combining (1) and (2),

$20\beta - 6\alpha = 9\alpha + 14\beta$

$\frac{\alpha}{\beta} = \frac{2}{5}$ i.e. $\alpha : \beta = 2 : 5$

12. C

Let $z = \frac{kx^3}{y^2}$ where k is a constant.

$3 = \frac{k(3)^3}{6^2}$

➔ $k = 4$

When $x = 5$ and $y = 2$,

$z = \frac{4(5)^3}{2^2}$

$= 125$

13. A

$a_4 = 2a_3 + a_2$

$= 2a_3 + 3$

$a_5 = 2a_4 + a_3$

$41 = 2(2a_3 + 3) + a_3$

$= 5a_3 + 6$

$a_3 = 7$

$a_4 = 2(7) + 3 = 17$

$\therefore\quad a_6 = 2a_5 + a_4$

$= 2(41) + 17$

$= 99$

14. A

$px + qy = 7$ i.e. $y = \dfrac{-p}{q}x + \dfrac{7}{q}$

Note that slope = $\dfrac{-p}{q}$ and the y-intercept = $\dfrac{7}{q}$

When $y = 0$, the x-intercept of $L = \dfrac{7}{p}$.

From the figure, $0 < \dfrac{7}{p} < 1$

➔ $p > 7$

∴ I is true.

From the figure,

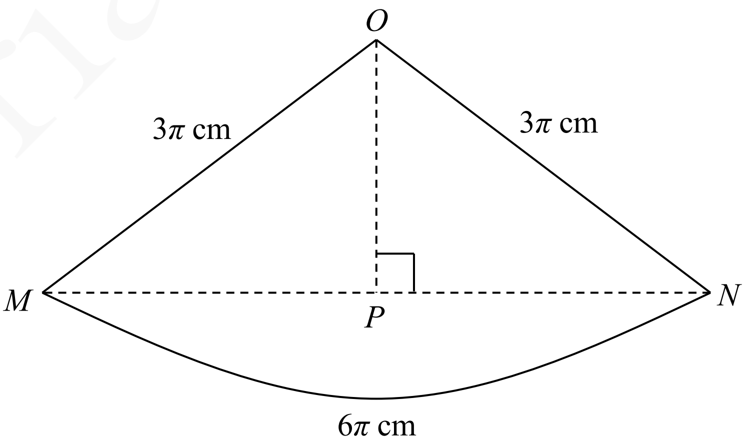
$1 < \dfrac{7}{q}$

➔ $q < 7 < p$

∴ II and III are NOT true.

15. D

Let θ be the angle at the centre of the sector OMN.



$2\pi(3\pi) \times \dfrac{\theta}{360^o} + 3\pi + 3\pi = 12\pi$

$\theta = \dfrac{360^o}{\pi} \approx 114.591559^\circ > 100^\circ$

∴ III is true.

The area of the sector OMN

$= \pi(3\pi)^2 \times \dfrac{360^o}{\pi} \times \dfrac{1}{360^o}$

$= 9\pi^2$

∴ I is true.

By cosine formula,

$MN^2 = (3\pi)^2 + (3\pi)^2 - 2(3\pi)(3\pi)\cos\dfrac{360^o}{\pi}$

$MN \approx 15.86135438$

The perimeter of ΔOMN

$\approx 15.86135438 + 3\pi + 3\pi$

≈ 34.71091031 cm < 35 cm

<u>Alternatively</u>

Let P be the foot of the perpendicular from O to MN.

$\angle MOP = \angle NOP = \dfrac{360^o}{2\pi}$

$MP = NP = 3\pi \sin\dfrac{360^o}{2\pi}$

The perimeter of ΔOMN

$= 2 \times 3\pi \sin\dfrac{360^o}{2\pi} + 3\pi + 3\pi$

≈ 34.71091031 cm < 35 cm

∴ II is true.

16. B

Let r cm be the radii of the right circular cylinder and the sphere respectively.

$2\pi r \times 35 + 2\pi r^2 = 492\pi$

$r^2 + 35r - 246 = 0$

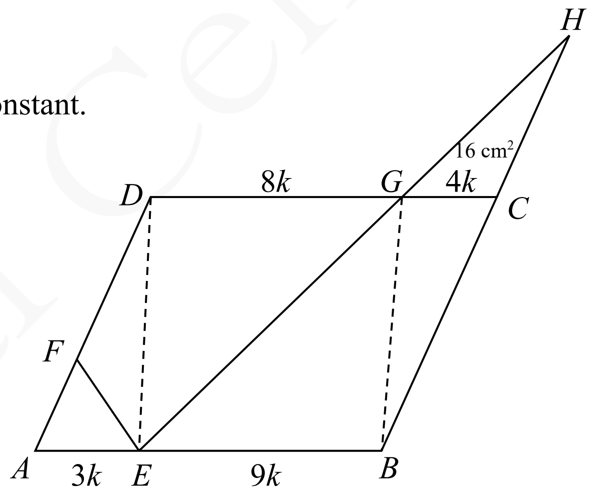
$r = 6$ or -41(rejected)

The volume of the sphere

$= \frac{4}{3}\pi(6)^3$

$= 288\pi$ cm^3

17. B



Note that $ABCD$ is a parallelogram with $AB = DC$.

$\because$ $AE : BE = 1 : 3$ and $CG : DG = 1 : 2$

$\therefore$ Let $AE = 3k$, $BE = 9k$, $CG = 4k$ and $DG = 8k$ where k is constant.

Note also that $\triangle HGC \sim \triangle HEB$.

$\frac{CG}{BE} = \frac{4}{9}$

$\frac{Area\ of\ \Delta CGH}{Area\ of\ \Delta BEH} = \left(\frac{CG}{BE}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

$\therefore$ Area of $\triangle BEH = 81$ cm^2

Area of $\triangle BEH$ = area of trapezium $CGEB$ + area of $\triangle CGH$

81 = area of trapezium $CGEB$ + 16

$\therefore$ Area of trapezium $CGEB = 65$ cm^2

Join DE and BG.

Note that $\triangle BGC$, $\triangle GEB$, $\triangle EDG$ and $\triangle DAE$ have the same heights.

$\therefore$ Area of $\triangle BGC$: Area of $\triangle GEB = CG : BE = 4 : 9$

$\because$ Area of trapezium $CGEB$ = area of $\triangle BGC$ + area of $\triangle GEB = 65$ cm^2

$\therefore$ Area of $\triangle BGC = 20$ cm^2 and area of $\triangle GEB = 45$ cm^2

Area of $\triangle EDG$: area of $\triangle BGC = DG : CG = 2 : 1$

$\therefore$ Area of $\triangle EDG = 40$ cm^2

Area of $\triangle DAE$: area of $\triangle GEB = AE : BE = 1 : 3$

$\therefore$ Area of $\triangle DAE = 15$ cm^2

Note that $\triangle EDF$ and $\triangle EAF$ have the same heights and

Area of $\triangle DAE$ = area of $\triangle EDF$ + area of $\triangle EAF$

$\therefore$ Area of $\triangle EDF$: Area of $\triangle EAF = DF : AF = 3 : 2$

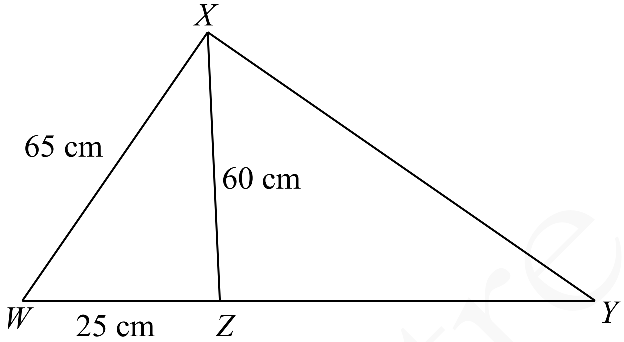
$\therefore$ Area of $\triangle EDF = 9$ cm^2 and area of $\triangle EAF = 6$ cm^2

Area of the quadrilateral $DFEG$ = area of $\triangle EDF$ + area of $\triangle EDG$

$= 9 + 40$

$= 49$ cm^2

18. C



$\because \quad \frac{WY}{XY} = \frac{XY}{YZ}$ and $\angle XYW = \angle ZYX$

$\therefore \quad \triangle WYX \sim \triangle XYZ$ (ratio of 2 sides, inc. $\angle$)

$\therefore \quad \frac{XY}{YZ} = \frac{WX}{XZ} = \frac{65}{60} = \frac{13}{12}$

i.e. $YZ = \frac{12}{13}XY$

$\because \quad XZ^2 + WZ^2 = 60^2 + 25^2 = 65^2 = WX^2$

$\therefore \quad \angle XZW$ is a right $\angle$.(Converse of Pythagoras' Theorem)

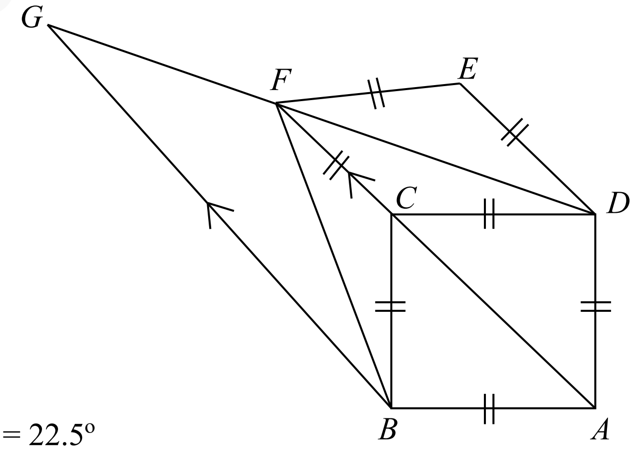
By Pythagoras' Theorem,

$XZ^2 + YZ^2 = XY^2$

$60^2 + (\frac{12}{13}XY)^2 = XY^2$

$XY = 156$ cm

19. D



$AB = BC = CD = DA$ (Property of sqaure)

$CD = DE = EF = FC$ and $\angle EFD = \angle EDF = \angle CFD = \angle CDF$ (Property of rhombus)

$\therefore \quad AB = BC = CD = DA = DE = EF = FC$

$\angle ACB = \angle ACD = 45^\circ$ (Property of square)

$\therefore \quad \angle FCB = \angle FCD = 135^\circ$ (adj. $\angle$s on st. line)

Note that $\triangle DEF \cong \triangle DCF \cong \triangle BCF$.

$\therefore \quad DF = BF$

$\angle CFB = \angle CBF$ (base $\angle$s, isos. $\triangle$)

$\angle FCB + \angle CFB + \angle CBF = 180^\circ$ ($\angle$ sum of $\triangle$)

$135^\circ + 2\angle CFB = 180^\circ$

$\angle CFB = 22.5^\circ$

$\therefore \quad \angle EFD = \angle EDF = \angle CFD = \angle CDF = \angle CBF = \angle CFB = 22.5^\circ$

$\angle FBG = \angle CFB = 22.5^\circ$ (alt. $\angle$s, $AF \parallel BG$)

$\angle FGB = \angle CFD = 22.5^\circ$ (corr. $\angle$s, $AF \parallel BG$)

$\therefore \quad BF = GF$ (sides opp. equal $\angle$s)

$\therefore \quad DF = FG$

$\therefore \quad$ I is true.

$\because \quad \angle FBG = \angle FGB = \angle EFD = \angle EDF = 22.5^\circ$

$\therefore \quad \triangle BFG \sim \triangle DEF$ (AA)

$\therefore \quad$ II is true.

$\angle ABG = \angle ABC + \angle CBF + \angle FBG = 90^\circ + 22.5^\circ + 22.5^\circ = 135^\circ$

$\angle BFD = \angle CFB + \angle CFD = 22.5^\circ + 22.5^\circ = 45^\circ$

$\angle ABG + \angle BFD = 135^\circ + 45^\circ = 180^\circ$

$\therefore \quad$ III is true.

20. D

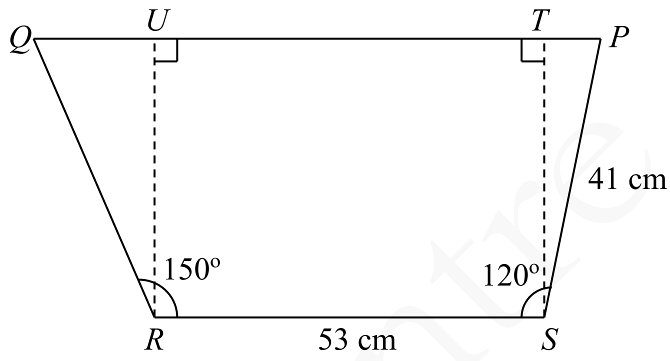
Let T and U be the feet of perpendiculars drawn from S and R to PQ respectively.

Note that $\angle SPT = 60^\circ$ and $\angle RQU = 30^\circ$

$TU = RS = 53$ cm

$PT = PS\cos\angle SPT = 41\cos 60^\circ$ cm

$RU = ST = PS\sin\angle SPT = 41\sin 60^\circ$ cm



$\frac{RU}{UQ} = \tan\angle RQU$

$UQ = \frac{RU}{\tan\angle RQU} = \frac{41\sin 60^o}{\tan 30^o}$ cm

$\therefore\quad PQ = PT + TU + UQ$

$= 41\cos 60^\circ + 53 + \frac{41\sin 60^o}{\tan 30^o}$

$= 135$ cm

21. B

Area of $\triangle ADE = \frac{1}{2} \times AE \times DE = 150$

i.e. $\frac{1}{2} \times 20 \times DE = 150$

➔ $DE = 15$ cm

By Pythagoras' Theorem,

$AD^2 = AE^2 + DE^2$

$AD^2 = 20^2 + 15^2$

$AD = 25$

Let F be the foot of perpendicular from E to AD. Then,

$\frac{1}{2} \times AD \times EF = 150$

$\frac{1}{2} \times 25 \times EF = 150$

➔ $EF = 12$

Let x cm be the perpendicular distance from E to CD.

By Pythagoras' Theorem,

$x^2 + EF^2 = DE^2$

$x^2 + 12^2 = 15^2$

$x = 9$

22. D

Let $\angle RUT = x$ and $\angle URT = \angle SRT = y$. Then,

$\angle VUR = \angle URT = y$ (alt. $\angle$s, RT // VU)

$\angle VUT + \angle VRT = 180^\circ$ (opp. $\angle$s, cyclic quad.)

i.e. $x + y + y + 33^\circ = 180^\circ$

$x + 2y = 147^\circ$ … (1)

$\angle RST + \angle RUT = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle RST + x = 180^\circ$

$\angle RST = 180^\circ - x$

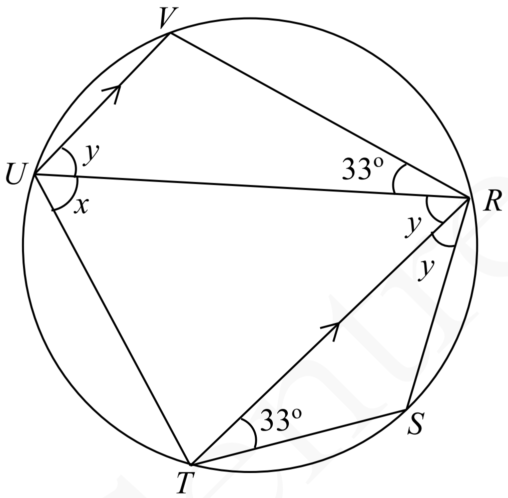
$\angle RST + \angle SRT + \angle RTS = 180^\circ$ ($\angle$ sum of $\triangle$)

$(180^\circ - x) + y + 33^\circ = 180^\circ$

$x - y = 33^\circ$ … (2)

Solving (1) and (2),

$x = 71^\circ$ and $y = 38^\circ$



23. B

Let $\angle ABC = x$ and $\angle ADC = y$. i.e. $x + y = 90^\circ$

$\angle CAD + \angle ADC + \angle ACD = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle CAD + y + 90^\circ = 180^\circ$

$\angle CAD = 90^\circ - y = x$

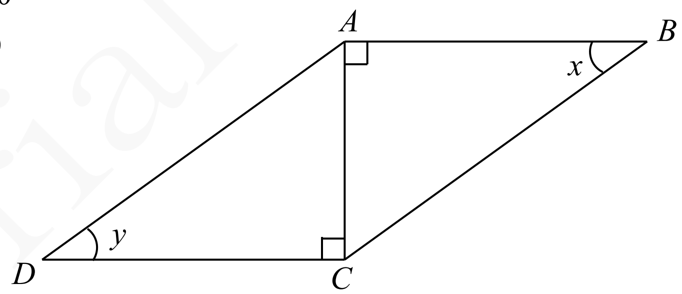
Similarly, $\angle ACB = y$

$\therefore$ $\triangle ABC \cong \triangle CAD$

$\tan \angle ACB$

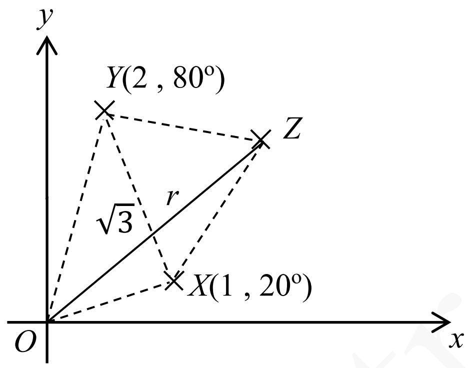
$= \dfrac{AB}{AC}$

$= \dfrac{AB}{CD}$



24. C

Note that $\angle XOY = 80^o - 20^o = 60^o$



By cosine formula,

$XY^2 = OX^2 + OY^2 - 2(OX)(OY)\cos\angle XOY$

$\quad = 1^2 + 2^2 - 2(1)(2)\cos 60^o$

$XY = \sqrt{3}$

$\therefore \quad XY = YZ = ZX = \sqrt{3}$

By sine formula,

$$\frac{\sin\angle OYX}{OX} = \frac{\sin\angle XOY}{XY}$$

$$\frac{\sin\angle OYX}{1} = \frac{\sin 60^o}{\sqrt{3}}$$

$\angle OYX = 30^o$

$\therefore \quad \angle OYZ = \angle OYX + \angle XYZ$

$\quad = 30^o + 60^o$

$\quad = 90^o$

By Pythagoras' Theorem,

$r^2 = OY^2 + YZ^2$

$\quad = 2^2 + (\sqrt{3})^2$

$r = \sqrt{7}$

25. B

Let $P = (x, y)$. Then,

$\sqrt{(x-a)^2 + (y-2a)^2} = \sqrt{a^2 + (2a)^2}$

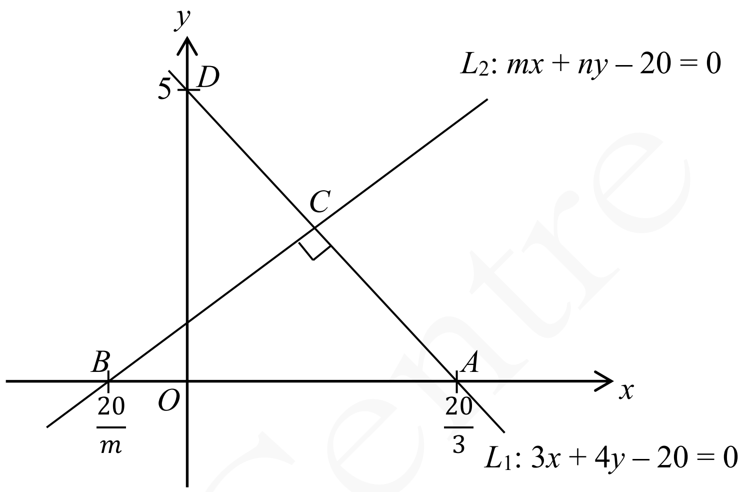
➔ $(x - a)^2 + (y - 2a)^2 = 5a^2$

$\therefore$ The locus of P is a circle centred at $(a, 2a)$ with a radius $\sqrt{5}a$.

26. B

Rewrite the equations of the straight lines in slope-intercept form.

$$\begin{cases} L_1 : y = -\frac{3}{4}x + 5 \text{ with } x\text{-intercept} = \frac{20}{3} \\ L_2 : y = -\frac{m}{n}x + \frac{20}{n} \text{ with } x\text{-intercept} = \frac{20}{m} \end{cases}$$



$\because \quad L_1 \perp L_2$

$-\frac{3}{4} \times -\frac{m}{n} = -1$ i.e. $n = -\frac{3}{4}m$

Let D be the y-intercept of L_1.

From the diagram shown on the right,

$AD = \sqrt{5^2 + \left(\frac{20}{3}\right)^2} = \frac{25}{3}$

Area of $\triangle ADO = \frac{1}{2} \times 5 \times \frac{20}{3} = \frac{50}{3}$

Note that $\triangle ABC \sim \triangle ADO$.

$\therefore \quad \left(\frac{AB}{AD}\right)^2 = \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADO}$

$\rightarrow \left(\frac{AB}{\frac{25}{3}}\right)^2 = \frac{6}{\frac{50}{3}}$

$AB = 5$

$\therefore \quad \frac{20}{3} - \frac{20}{m} = 5$

$m = 12$

$n = -\frac{3}{4}m$

$= -\frac{3}{4}(12)$

$= -9$

27. A

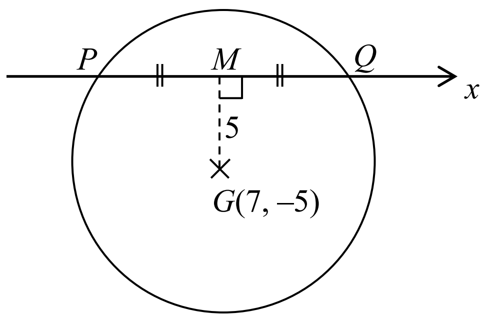
Let M and G be the mid-point of PQ and the centre of the circle respectively. Then,

$MG = 5$ and $PM = MQ = 12$

Radius, $r = \sqrt{5^2 + 12^2} = 13$

The equation of C is $(x-7)^2 + [y-(-5)]^2 = 13^2$

i.e. $x^2 + y^2 - 14x + 10y - 95 = 0$.



28. C

The expected number of tokens

= P("2") $\times$ 10 + P("3") $\times$ 15 + P("4") $\times$ 25 + P("5") $\times$ 50

$= \frac{3}{6} \times 10 + \frac{1}{6} \times 15 + \frac{1}{6} \times 25 + \frac{1}{6} \times 50$

= 20

29. A

From the bar chart, $Q_1 = 4$, $Q_3 = 5$

Inter-quartile range $= Q_3 - Q_1$

$= 5 - 4$

$= 1$

30. B

$\frac{\alpha+\beta-4-3+1+1+1+4}{8} = 0$

$\therefore$ $\alpha + \beta = 0$

$\therefore$ III is true.

$\because$ $\alpha \neq \beta$

$\therefore$ Mode = 1

$\therefore$ I is true.

$\because$ $\alpha = -\beta$ and the range is 10

$\therefore$ α and β must be 5 and –5.

Rearrange the data in order: –5, –4, –3, 1, 1, 1, 4, 5

$\therefore$ Median $= \frac{1+1}{2} = 1$

$\therefore$ II is not true.

31. D

$3E000000000000_{16}$

$= 3 \times 16^{13} + 14 \times 16^{12}$

$= (2 + 1) \times (2^4)^{13} + (2^3 + 2^2 + 2) \times (2^4)^{12}$

$= (2 + 1) \times 2^{52} + (2^3 + 2^2 + 2) \times 2^{48}$

$= 2^{53} + 2^{52} + 2^{51} + 2^{50} + 2^{49}$

32. C

$p^2 - 4q^2 = (p + 2q)(p - 2q)$

$p^3 - 8q^3 = (p - 2q)(p^2 + 2pq + 4q^2)$

$(p + 2q)(p^2 - 4q^2) = (p + 2q)(p + 2q)(p - 2q) = (p + 2q)^2(p - 2q)$

$\therefore$ L.C.M. $= (p + 2q)^2(p - 2q)(p^2 + 2pq + 4q^2)$

$= (p + 2q)^2(p^3 - 8q^3)$

33. C

$\because$ Intercept on the vertical axis = 12

$\therefore \begin{cases} \log_{25}x = 0 & \text{i.e. } x = 1 \\ \log_5 y = 12 & \text{i.e. } y = 5^{12} \end{cases}$

We have $5^{12} = m(1)^n$

$\Rightarrow$ $m = 5^{12}$

$\because$ Intercept on the horizontal axis = 2

$\therefore \begin{cases} \log_{25}x = 2 & \text{i.e. } x = 25^2 \\ \log_5 y = 0 & \text{i.e. } y = 1 \end{cases}$

We have $1 = 5^{12}(25^2)^n$

$= 5^{12}(5^2)^{2n}$

$\Rightarrow$ $12 + 4n = 0$

$\Rightarrow$ $n = -3$

Alternatively

Slope of the straight line = $\frac{0-12}{6-0}$ = –6

$\therefore$ $\log_5 y = -6\log_{25}x + 12$

$= \log_{25}x^{-6} + \log_{25}25^{12}$

$= \log_{25}(25^{12}x^{-6})$

$\frac{\log y}{\log 5} = \frac{\log(25^{12}x^{-6})}{\log 25}$

$\log y = \frac{\log(25^{12}x^{-6})}{2}$

$= \log\ (25^{12}x^{-6})^{\frac{1}{2}}$

$= \log 25^6x^{-3}$

$\therefore$ $y = 25^6x^{-3}$

$\therefore$ $n = -3$

34. D

For $a > 1$, the graphs of $y = \log_a x$ and $y = a^x$ do not intersect.

$\therefore$ $0 < a < 1$

$\therefore$ I is true.

Note that $\log_a OQ = 0$

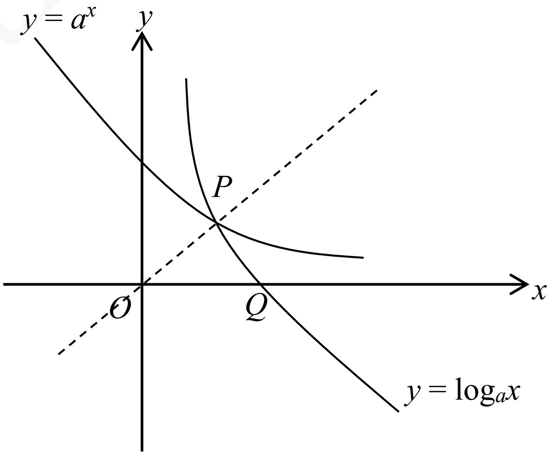
$OQ = 1 > a$

$\therefore$ II is true.

The graphs of $y = \log_a x$ and $y = a^x$ are symmetrical about the straight line $y = x$.

$\therefore$ $\angle POQ = 45°$

$\therefore$ III is true.



35. A

$i^9 + i^{10} + i^{11} + \ldots + i^{999}$

$= \frac{i^9(1-i^{991})}{1-i}$

$= \frac{i[1-(-i)]}{1-i}$

$= \frac{i(1+i)(1+i)}{(1-i)(1+i)}$

$= \frac{i(1+2i+i^2)}{1+1}$

$= -1$

Alternatively

$i^9 + i^{10} + i^{11} + \ldots + i^{999}$

$= i^8(i + i^2 + i^3 + \ldots + i^{991})$

$= (i - 1 - i + 1) + (i - 1 - i + 1) + \ldots + (i - 1 - i)$

$= -1$

36. B

$$\begin{cases} x = 11 \ \ldots (1) \\ 4x + 5y - 19 = 0 \ \ldots (2) \\ 7x - 6y + 11 = 0 \ \ldots (3) \end{cases}$$

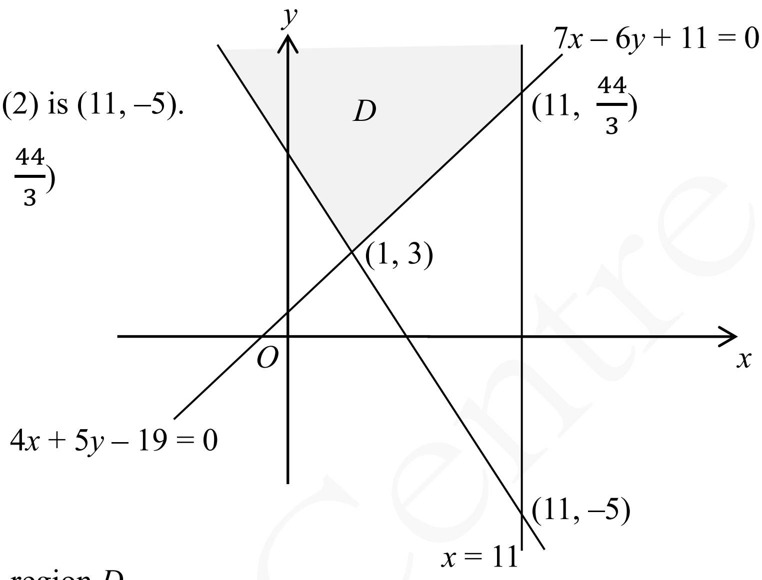
Solving (1) and (2), the intersection of (1) and (2) is $(11, -5)$.

Similarly, the intersection of (1) and (3) is $(11, \frac{44}{3})$

while that of (2) and (3) is $(1, 3)$.

Let $P(x, y) = 8x - 6y + 11$

Only $(11, \frac{44}{3})$ and $(1, 3)$ fall in region D.



$P(11, \frac{44}{3}) = 8(11) - 6\left(\frac{44}{3}\right) + 11 = 11$

$P(1, 3) = 8(1) - 6(3) + 11 = 1$

$\therefore$ The greatest value of $8x - 6y + 11$ is 11 in region D.

37. A

Let d be the common difference of the arithmetic sequence. Then,

$q = p + d$ and $r = p + 2d$

$3^q = 3^{p+d} = 3^d \cdot 3^p$

$3^r = 3^{p+2d} = (3^d)^2 \cdot 3^p$

$\therefore$ 3^p, 3^q and 3^r is a geometric sequence with common ratio 3^d.

$\therefore$ I is true.

$$\frac{5}{q} \div \frac{5}{p} = \frac{p}{q} = \frac{p}{p+d}$$

$$\frac{5}{r} \div \frac{5}{q} = \frac{q}{r} = \frac{p+d}{p+2d} \neq \frac{p}{p+d}$$

$\therefore$ $\frac{5}{p}, \frac{5}{q}, \frac{5}{r}$ is NOT a geometric sequence.

$\therefore$ II is NOT true.

$(q - r) - (p - q) = [(p + d) - (p + 2d)] - [p - (p + d)] = 0$

$(r - p) - (q - r) = [(p + 2d) - p] - [(p + d) - (p + 2d)] = 3d$

$\therefore$ $p - q, q - r, r - p$ is NOT an arithmetic sequence.

$\therefore$ III is NOT true.

38. A

Join AD.

$\angle ADC = 90^\circ$ ($\angle$ in semi-circle)

$\angle DAC = \angle CDT = 41^\circ$ ($\angle$ in alt. segment)

$\angle ACD + \angle DAC + \angle ADC = 180^\circ$ ($\angle$ sum of Δ)

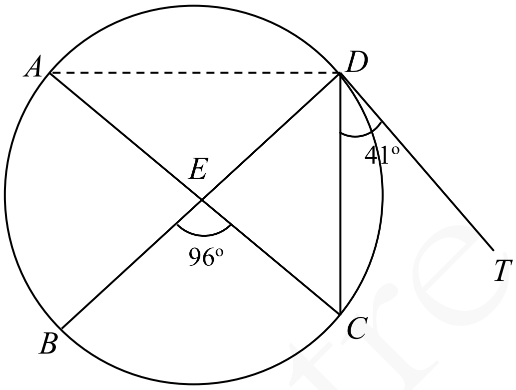
$\angle ACD + 41^\circ + 90^\circ = 180^\circ$

$\angle ACD = 49^\circ$

$\angle CDE + \angle ACD = \angle BEC$ (ext. $\angle$ of Δ)

$\angle CDE + 49^\circ = 96^\circ$

$\angle CDE = 47^\circ$



39. B

$\tan^3\theta = 2\tan\theta$

$\tan\theta(\tan^2\theta - 2) = 0$

$\tan\theta = 0$ or $\tan^2\theta = 2$

$\tan\theta = 0$ or $\tan\theta = \sqrt{2}$ or $\tan\theta = -\sqrt{2}$

$\theta = 180^\circ$, 125° or 235° for $90^\circ < \theta < 270^\circ$.

40. B

Let the length of each side of $PQRS$ be $2x$. Let T be the foot of perpendicular from P to plane QRS. The required angle is $\angle PQT$.

Let V be the foot of perpendicular from T to QS. By symmetry,

$QV = VS = x$ and $\angle TQV = \angle TQR = 30^\circ$. Then,

$\frac{x}{TQ} = \cos 30^\circ$

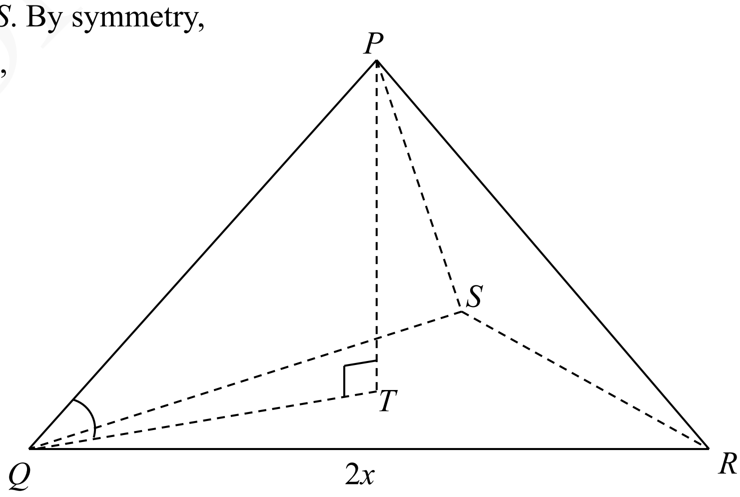
$TQ = \frac{x}{\cos 30^o}$ i.e. $TQ = \frac{2x}{\sqrt{3}}$

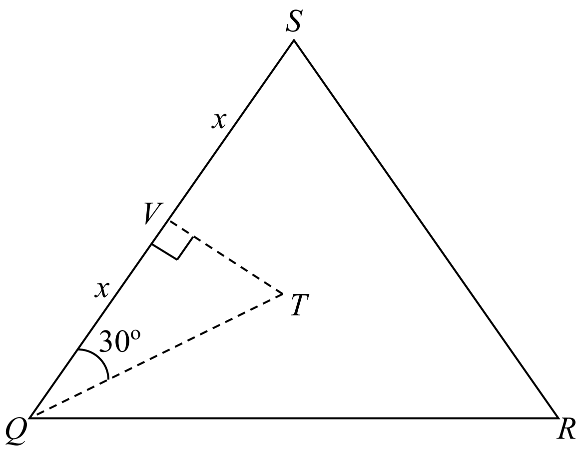
$\cos\angle PQT = \frac{TT}{PQ}$

$= \frac{\frac{2x}{\sqrt{3}}}{2x}$

$= \frac{1}{\sqrt{3}}$

$\angle PQT = 55^\circ$ (correct to the neanest degree)





41. C

Let ABC be the inscribed circle in ΔOUV where A, B, C are tangent points as shown below.

Note that $VA = VB$ and $UB = UC = 14$.

Let $VA = VB = k$.

By Pythagoras' Theorem,

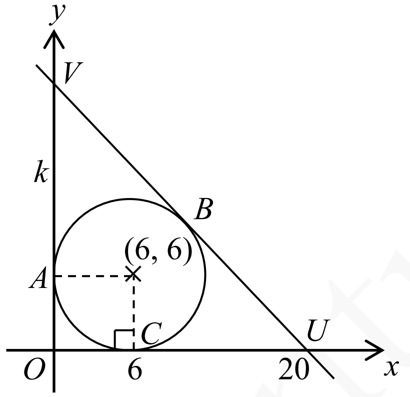
$(k + 6)^2 + 20^2 = (k + 14)^2$

➔ $k = 15$

∴ Area of ΔOUV

$= \frac{(15+6)\times 20}{2}$

$= 210$



42. B

Number of working groups required

= Number of all possible groups – number of groups without managers

$= C_7^{18} - C_7^{16}$

= 20 384

43. C

The required probability

= 1 – P("4 cans of grape juice")

$= 1 - \frac{C_2^9}{C_6^{13}}$

$= \frac{140}{143}$

44. A

Let b and g be the test scores of the boy and the girl respectively. Let μ be the mean of the test scores.

$\frac{b-\mu}{2} = -2$ i.e. $b = \mu - 4$

$\frac{g-\mu}{2} = z$ i.e. $g = \mu + 2z$

∴ $\mu - 4 - (\mu + 2z) = 6$ or $\mu + 2z - (\mu - 4) = 6$

$z = -5$ or 1

45. D

$m_1 = \frac{a+b+c+d}{4}$ i.e. $a + b + c + d = 4m_1$

$m_2 = \frac{2a+2b+2c+2d}{4} = 2m_1$

$m_3 = m_1 + 3$

$\therefore$ $m_1 + m_3 = m_1 + m_1 + 3 = 2m_1 + 3 > 2m_1 = m_2$

$\therefore$ I is true.

Without loss of generality, let $d > c > b > a$. Then,

$r_1 = d - a$

$r_2 = 2d - 2a = 2r_1$ and $r_3 = r_1$

$\therefore$ $r_1 + r_3 = 2r_1 = r_2$

$\therefore$ II is true.

Note that $v_1 = v_3$.

$v_2 = 2^2 v_1 = 4v_1$

$v_1 + v_3 = 2v_1 < v_2$

$\therefore$ III is true.